Procurement Mechanisms in the Presence of Learning by Doing

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Procurement Mechanisms
in the Presence of Learning by Doing

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To my husband, for his love and patience.

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SUMMARY

In markets where suppliers experience learning by doing over time or, more generally, economies of scale in production, buyers are auctioning off longer-term contracts with an eroding price policy. Under an eroding price contract, the buyer initially competitively awards production to the lowest-bid supplier via an auction. Before the auction takes place, the buyer makes it clear to the suppliers that, if chosen, a sequence of price reductions will be mandatory in subsequent periods.

In this thesis, we mainly study the design of the optimal eroding price contract in a two period setting under three different model settings : (1) Every supplier faces a *new cost* in each period (NLI model : chapter 3), (2) The supplier who wins the auction in the first period *locks-in* his cost for the future, and the buyer makes the future payment based on the winning supplier's current *bid* (LI1 : chapter 4), and (3) The supplier who wins the auction in the first period *locks-in* his cost for the future, and the buyer makes the future payment based on the winning supplier's *actual cost* (LI1 : chapter 5). Under NLI setting, the magnitude of the cost reduction due to learning by doing is common knowledge, while the magnitude is uncertain under LI1 and LI2 settings. We also study the optimal reserve prices in sequential independent auctions under NLI setting (chapter 6).

We go on to compare the performance of the eroding price policy against sequential independent auctions (without or with the optimal reserve prices) under the above model settings. Via analytical and numerical comparisons, we find that even in the presence of learning by doing/economies of scale in production, a buyer is often better off running sequential auctions with a reserve price, rather than limiting competition and contracting with a single supplier in the hopes of extracting a better future price.

CHAPTER I

INTRODUCTION

Since the concept of an “online procurement (reverse) auction” emerged in the mid-1990’s, it has become an increasingly popular and powerful tool for outsourcing. Success stories at Hewlett-Packard(Mohara et al.[32]), Motorola(Arensman[3], Minahan[31], Metty et al.[30]), and Volkswagon (Beall et al.[6]), to name a few, have grabbed the attention of procurement managers across industries attempting to identify ways to reduce costs. One clear lesson that has emerged from the use of auctions in a variety of markets is that ‘one size does not fit all’. Auction makers must take into account the characteristics of the market in designing their auction.

For example, in the automotive industry, which has been an early adopter of B2B auctions, it is very common that each customer has his own design specifications even for pseudo-commodity products, such as leather car seats. Under such settings, a supplier frequently can not use the same production line for two different customers due to different design engineering spec, e.g., colors, leather quality, number of wrinkles, etc. As a result, the supplier must incur a buyer-specific set-up cost to design the production line to meet the customer’s specific demands. In addition, as the supplier (and often the buyer) learns more about the buyer’s likes/dislikes as well as the process of the production, he is able to produce the good more efficiently and cost effectively. In markets such as this, where suppliers experience learning by doing over time or, more generally, economies of scale in production, buyers are auctioning off longer-term contracts with an eroding price (a.k.a. cascading price) policy (see table 1.1 for examples). Under an eroding price contract, a buyer awards a multi-period contract where the price paid to the supplier declines over time at a prespecified rate. The elements of a long-term contract, where a supplier is guaranteed a buyer’s business, or is at least given the right of first refusal, combined with a pre-specified declining price are thought to be effective in identifying a win-win situation for both the buyer and winning supplier.

Table 1.1: Examples of Commodities auctioned via eroding price contract

Category	Commodity
Electronics	PCA/PCBs, cables/assemblies, capacitors, transformers/transducers, wire harnesses monitors, sockets, transceivers
Engineering & Construction	compressor reblading
Metals	fasteners, fabrication/stamping, flanges, regulators, steel tubes, valve assembly high volumed machined parts,
MRO & Services	contact center, copiers, janitor services, office supplies, temporary labor services, branch automation deployment, uniforms
Paper & Packaging	bulk bags, corrugated packaging, labels folding cartons
Plastics	decorative plastics, molded and die cut EPS, vertical injection molding closures, seals/Gaskets
Raw Materials	molded/extruded rubber components

While the benefits and advantage of such a procurement mechanism may at first seem intuitive, we are unaware of any work that has (*i*) studied the optimal design of the eroding price contract and/or (*ii*) demonstrated that an eroding price contract is more cost effective than alternative popular procurement methods. This thesis mainly aims to further our understanding of the appropriateness of eroding price contracts under various market settings. We assume that the buyer faces the same N potential suppliers in each period, and compare the performance of eroding price contracts (EPC) against sequential independent auctions (SI) under two cost frameworks,

• No-Lock-In (NLI): Every supplier faces a new cost in each period. This assumption is reasonable when the duration between bidding periods is long or technology and industry environment changes fast (e.g., the electronics industry); under such a scenario a supplier may experience changes in its own subcontractors, price of raw materials, financial situation, etc, and find itself with altered costs in the next bidding round.

• Lock-In (LI): The supplier who wins the auction at $T = 1$ 'locks-in' his (baseline) cost for the future; all other suppliers redraw their costs at $T = 2$. This setting reflects a situation where the winning supplier is able to use his newly won contract to write a contract to lock-in his own input prices.[1]

Besides the design of the eroding price contract and its performance, we explore characterizing the optimal reserve prices in sequential auctions. By this, we can evaluate the optimal eroding price contract by comparing with the sequential auctions with the optimal reserve prices.

Chapter 1.1 gives an overview of the relevant literature; we discuss our models and assumptions in chapter 2 . Using game-theoretic methods, we study the equilibrium behavior of suppliers under EPC and SI, and compare their performance under the NLI setting (chapter 3) and LI settings (chapter 4 and 5). In chapter 6, we study the optimal reserve prices under SI (as a complementary of the optimal eroding price schedule under EPC). We conclude our thesis with future directions for research in chapter 7.

1.1 Literature Review

Learning by doing can be more generally interpreted as a positive synergy (or complementarity) over a bundle of goods, where the cost of a bundle of goods is less than the sum of the costs of the goods individually. Von der Fehr and Riis[43], Grimm[15], Jeitschko and Wolfstetter[16], Menezes and Monteiro[29], Sorensen[40] study how the presence of synergies impacts equilibrium bidding behavior and the auctioneer's profit. Similar to the model we adopt, they allow bidders to supply more than one unit and assume that the winner

[1] Under symmetric and strictly monotonically increasing bid functions at $T = 1$, the incumbent supplier would always be the least cost supplier at $T = 2$ if suppliers who lost at $T = 1$ kept the same costs across both periods; hence, we ignore this simple case.

in the first auction has a cost advantage over the losers in the subsequent auction. All, with the exception of Von der Fehr and Riis[43], assume a NLI cost setting, and hence assume that the costs in the first and the subsequent auction are independent. Von der Fehr and Riis[43] and Jeitschko and Wolfstetter[16] study the effect of the future market opportunities on the bidding behavior and the auctioneer's selling price in the first and the subsequent auctions, while Grimm[15] and Menezes and Monteiro[29] compare the performance of a bundle auction with sequential independent auctions. Sorensen[40] suggests the methodology to design the optimal mechanism in such a setting (See table 1.2 for detailed model settings of those papers). Our work deviates from their studies in two important ways: We introduce and study the optimal design of eroding price contracts[2] and extend the analysis of eroding price contracts and sequential independent auctions to a LI setting; Von der Fehr and Riis[43] also considers this case, but focus only on sequential auctions.

Motivated by the Federal Communication Commission's sale of spectrum bandwidth via simultaneous ascending auctions, Krishna and Rosenthal[20] study the bidding behavior of global and local bidders in a sequential multi-unit 2^{nd} price auction, where a local bidder has positive value for only one (particular) object while a global bidder has positive synergies associated with winning more than one object. the global bidder's valuation from winning only one object is x, while his value from winning two objects is $2x+\alpha$, where x is i.i.d. and α is a known constant shared by all the global bidders. A key characteristic of these papers is that local bidders participate in only one of the auctions, i.e., the global bidder faces a new set of locals in each auction and it is a (weakly) dominant strategy for a local to bid his true valuation in each auction. Elmaghraby[13]; she considers an extension of Krishna and Rosenthal[20] where the same set of local bidders can participate in both auctions. In contrast, this thesis considers a setting where all of bidders are global bidders, and hence

[2]There are literatures which also have studied the design of the optimal two-periods contracts in different stream rather than auction theory. Baron and Besanko[5] and Courty and Li[9] share a similar feature with our eroding price contract in the sense that agents learn their costs(or values) over time, but the contracts are signed at $T = 1$; Baron and Besanko[5] study the optimal "*regulatory*" policy where firms knows only their costs at $T = 1$ at the timing of the contract, and later they learn costs at $T = 2$ (i.e., the costs may independent or correlated), and Courty and Li[9] study the optimal "*price discrimination*" for a monopolist where consumers consumers learn their actual values after the contract (i.e., they only knows the distribution of their values at $T = 1$).

Table 1.2: Literatures on Sequential Auctions with Synergies

Literature	Value Distribution of bidders (2-periods)	Synergy of having two objects	Focus in studies
Menezes & Monteiro (1999)	Same values at $T = 1$ & 2	$d(v)^{\S} = v_1 + v_2 + \alpha$: $\alpha > (<)0$	Price trends in SI Comparisons between BA and SI
Jeitschko & Wolfstetter (2002)	2-points distribution, $\{0, v\}$ same at $T = 1$ & 2	Incumbent & entrant have different probability of having v	Price trends in SI Comparison between first- and second-price auction
Vonder Fehr & Riis (2003)	Distribution at $T = 2$ is conditional on the value at $T = 1$	Incumbent or entrants retain their value at $T = 2$	Price trends in SI
Grimm (2003)	Incumbent & entrants redraw their costs at $T = 2$ $F_E(v) \prec_{FSD} F_I(v)$	$d(v) = v_1 + v_2 + \alpha$: $\alpha > 0$	Comparisons between BA and SI (in procurement)
Sorensen (2003)	Incumbent & entrants redraw their costs at $T = 2$	$d(v) = v_1 + v_2 + \alpha$: $\alpha > 0$	Optimal SI

§ $d(v)$: utility of owing two objects of a bidder
Incumbent : a winner at $T = 1$
Entrant : a loser at $T = 1$ or new participant at $T = 2$
BA : Bundle auction
SI : Sequential Independent Auction

all of the bidders can participate in both auctions, although only one will experience a cost reduction.

A growing number of papers consider the design of procurement strategies in a dynamic setting (please see Elmaghraby[11] for an overview of the literature). Many of the papers that have studied repeated procurements are mainly concentrated on the Defense of Department (DoD) procurement. DoD procurement is often comprised of multiple stages, encompassing concept design, R&D, initial production, and full production, and typically any cost reductions brought about by the incumbent are transferred to any new suppliers.

Rob[37], Anton and Yao[1], Riordan and Sappington[36], and Laffont and Tirole[22] address the government's procurement strategy and discuss how the buyer can extract optimal effort from the developer (i.e., the selected supplier) in the R&D stage under asymmetric information and experience (i.e., only the supplier knows his cost). In contrast, we focus on repeat procurement in a setting where the product(s) to be supplied are well-defined (i.e., the buyer is not concerned with R&D) and where the learning advantage of the existing supplier is not transferable to other suppliers.

Motivated by DoD procurement (but not limited to it), however, Lewis and Yildirim[25] study the optimal procurement design when the buyer procures from one of two suppliers in the infinite time horizon under the short term commitment. Each supplier has his finite learning curve and he experiences the cost reduction during his learning curve. Under such setting, the buyer balances the cost reduction from learning by doing against reduction of competition if one supplier become dominant in the market by solving Markov-Perfect equilibrium. They find that the buyer tends to switches suppliers when the learning economies are small, and the rate of the learning is too slow due to the buyer's preference for balanced competition among suppliers. Again, their main focus is limited to the short term contract.

Klotz and Chatterjee[19] study the design of two sequential auctions when the buyer wishes to dual sourcing (there are only two potential suppliers in this market and the buyer wishes to purchase from both). Both suppliers incur a cost for submitting a bid, and experience learning by doing cost reductions (depending on the quantity produced). To induce competition in each period, the buyer guarantees each supplier a portion of her business in the first period, and allow the suppliers to compete for the remainder. They conclude that dual sourcing is more effective when the buyer can not commit to a long term contract or the bidders can not buy-in to the future gain.

Under a static framework which does not consider possible shifts in supplier costs due to learning by doing/economies of scale, Bulow and Klemperer[8] argue that a buyer is always better off maximizing the number of bidders (Corollary 1, pg. 189) and hence a buyer should not favor procurement mechanisms where she 'locks-in' to a single supplier in return for the supplier offering her a more competitive price. They consider a framework where a seller faces $N+1$ buyers; the bidders' valuations can be either independent or affiliated (but are symmetric). Bulow and Klemperer compare the performance of two selling mechanisms; (i) an English auction with N bidders and a take-it-or-leave-it price offered to the last remaining bidder (where the take-it-or-leave-it price is determined after the seller has had the opportunity to gather information from the bidding process) and (ii) an English auction with one more bidder, $N+1$, but without any additional negotiation stage. Under this setting, they prove that one additional bidder always yields a higher expected revenue to the seller than any negotiation process with one less bidder. This thesis aims to answer if Bulow and Klemperer[8]'s result carries over to a dynamic setting with synergies.

CHAPTER II

MODEL

A buyer wishes to procure Q units of a good in each of two time periods. The buyer wishes to sole source in each period, and incurs a switching cost of s if she switches suppliers at $T = 2$. A switching cost may occur due to the cost of running a new auction, training the new supplier about characteristics specific to that buyer and/or the cost of moving parts or changing tooling from the incumbent supplier to a new supplier. The buyer faces $N \geq 3$ potential suppliers: We assume that all N suppliers have been pre-qualified to participate in the auction, i.e., they all adequately meet the buyer's demand specifications, and have sufficient production capacity to fill the buyer's entire order. Thus, the buyer's sole criterion when selecting a supplier is price. Given the suppliers' adequate capacity, we can assume without loss of generality, that $Q = 1$.

A supplier t's cost at time T is given by t^T, for $T = 1, 2$. We assume that each bidder's cost at time T is private information and is drawn independently from a continuous and differentiable distribution. At $T = 1$, each bidder's cost is drawn from the *same* distribution $F(t^1) : F(C) = 0, F(C+1) = 1, C > 0$ with a corresponding density $f(t^1)$. We assume that $f(t^1)$ is strictly positive anywhere on $[C, C+1]$.

A supplier who is not selected at $T = 1$ is referred to as an *entrant* at $T = 2$, and redraws his (privately known) cost at $T = 2$, t_E^2 from the same (original) common distribution function F (see figure 2.1). We refer to the supplier who fulfills the buyer's demand at $T = 1$ as the *incumbent* supplier. After supplying the buyer for one period, the incumbent supplier experiences "learning by doing", which translates into a reduction in his production cost at $T = 2$. We assume that the maximum possible amount of cost reduction is same for all bidder types and given by α (we consider relaxing this assumption in chapter 4.4.2). α is assumed to be small relative to C so that $C - \alpha \geq 0$ (without loss of generality, we assume that $C \geq 1$, and α is a fraction of 1). We define the incumbent's cost at $T = 2$ as

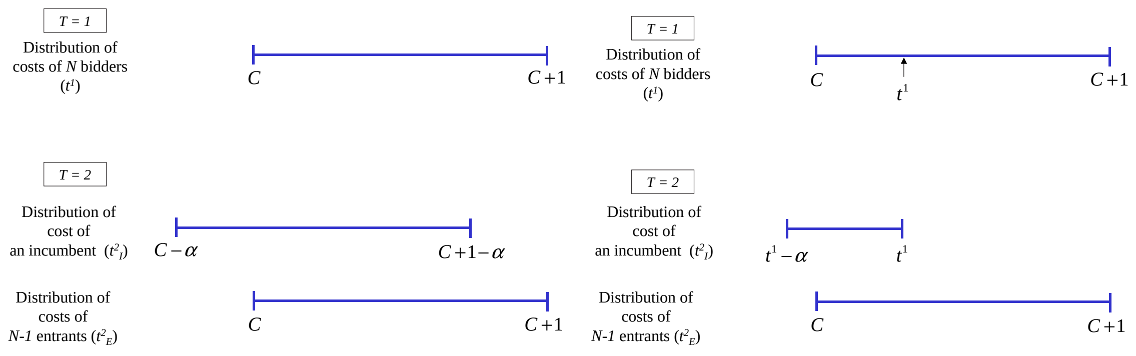


Figure 2.1: Costs at $T = 1$ and $T = 2$

t_I^2. In this thesis, we study two different cost structures for the incumbent: (1) Under the No-Lock-In (NLI) setting, the incumbent's cost at $T = 2$, t_I^2, is drawn from the distribution $F_I(t_I^2) : F(C-\alpha) = 0, F(C+1-\alpha) = 1$, which shifts downward by α from F. (2) Under the Lock-In (LI) setting, the incumbent's cost at $T = 2$ is drawn from the distribution $F_I(t_I^2) : F(t^1 - \alpha) = 0, F(t^1) = 1$ (see figure 2.1).

We can consider that the incumbent's cost at T=2 is composed of two elements : his baseline production cost and learning effect cost. Under NLI setting, the magnitude of the learning effect cost is a common knowledge ($= \alpha$), while the transitory cost at $T = 2$ is uncertain (i.e., drawn from the distribution F). In contrast, under LI setting, the magnitude of the learning effect is uncertain (i.e., drawn from range of $[0, \alpha]$), while his baseline production cost is known by the first auction ($= t^1$). That is, the supplier locks-in his transitory cost, but possibly can reduce his cost from learning by doing effect which is uncertain at $T = 1$.

It is important to emphasize that this type of cost reduction occurs as a result of production (e.g., worker familiarity and experience with the production process) during $T = 1$ and is not the result of any cost-reducing measures that the supplier may undertake, such as upgrading equipment, educating workers, etc. The potential for learning by doing can be quite substantial in some industries: NASA hosts a learning curve calculator http://www.jsc.nasa.gov/bu2/learn.html, and publishes statistics that estimate the

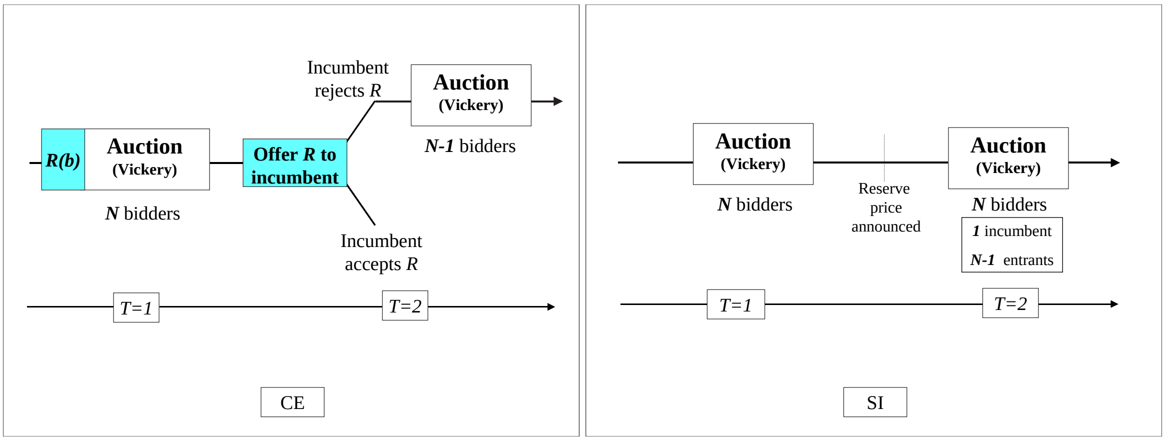


Figure 2.2: Procurement Mechanisms

cost savings arising from climbing the learning curve to be 15% in aerospace, up to 20% in shipbuilding and up to 25% in repetitive electrical operations.

We assume that all participants are risk-neutral and maximize(minimize) their expected profits(costs) and that N is common knowledge. Furthermore, we assume that suppliers behave rationally and they take into account expected future profit streams when determining their optimal bids at $T = 1$. The buyer (credibly) commits herself in advance to a procurement mechanism, i.e., a set of allocation policies and payment contracts. In this thesis, we describe and compare two different types of procurement mechanisms that are currently being used in B2B auctions: (1) an Eroding price contract with conditional commitment (EPC), and (2) Sequential independent auctions (SI). Without loss of generality, we focus on direct mechanisms, whereby suppliers bid by reporting a cost(type), which may differ from their true cost. The winner is selected based on the reported cost and the buyer uses the suppliers' reported types in designing her optimal eroding price schedule.

2.1 Procurement Mechanisms

Mechanism 1: Eroding Price Contract (EPC) (see figure 2.2) When it is common knowledge that a supplier may experience some learning by doing and hence a reduction in production costs at $T = 2$, some buyers wish to use a long-term contract to exert some control over the price that they pay at $T = 2$ and ask their suppliers to adhere to an eroding price contract. Under an eroding price contract, the buyer holds an auction at $T = 1$ to select a supplier and each supplier simultaneously submits a bid indicating the amount he wishes to be paid per unit at $T = 1$. The lowest bidding supplier is awarded the buyers' business at $T = 1$ and is paid the lowest rejected price (Vickrey auction)[1]. However, the buyer announces upfront a price schedule to which the winning supplier must adhere. That is, before the auction at $T = 1$, the buyer announces an eroding price schedule $R(B^1_{EPC}(t^1))$ which states, for each submitted bid at time $T = 1$, the price the supplier will be paid at $T = 2$. Under strictly monotonically increasing bid functions, Myerson's direct revelation principle implies that we can model the eroding price schedule as a function of a supplier's true type, i.e., $R(B^1_{EPC}(t^1)) = R(t^1))$. We assume that $R(t^1)$ is in the range of the incumbent's cost, i.e., $R(t^1) \in [C - \alpha, C + 1 - \alpha]$ $\forall$ t^1 under NLI, and $R(t^1) \in [t^1 - \alpha, t^1]$ $\forall$ t^1 under LI. This implies that the buyer does not consider prices that are guaranteed to make the supplier unprofitable at $T = 2$ or pay the supplier more than his possible highest cost.

Under EPC, the incumbent supplier is given the *option* to reject $R(t^1)$ *after* he has observed his cost at $T = 2$. If the incumbent decides to reject $R(t^1)$, then the buyer holds a second (Vickrey) auction with the remaining $N - 1$ suppliers at $T = 2$, where the incumbent is not invited to participate in the second auction. The rationale for excluding the incumbent is as follows: If the incumbent cannot participate in a second auction, then it is optimal for him to accept $R(t^1)$ if $R(t^1) > t^2_I$ and reject it otherwise. If the incumbent were not excluded from the second auction, then he would have the incentive to trade-off his profit under $R(t^1)$ with his expected profit from participating in the second auction, and may not accept $R(t^1)$ even if it is profitable. Under EPC, each supplier has two decision variables; his bid at $T = 1$ and his bid at $T = 2$ if he should lose at $T = 1$ and the incumbent should reject the contract at $T = 2$.

[1]The most common auction format adopted by FreeMarkets is an open descending price auction. Under this format, the winning bidder is paid the price at which his last opponent drops out of the auction. It is well-known that a Vickrey auction and an open-auction yield the same outcome in the single unit case (Myerson[34]). We use a Vickrey auction as a stylistic representation of the open ascending auction format, with the acknowledgment that the two formats *may* not lead to the same equilibrium bidding behavior under our modeling assumptions. The strategic and revenue equivalence of the two auction formats remains an open research question.

Mechanisms 2 : Sequential Independent Auctions (SI) (see figure 2.2) Under SI auctions, the buyer prefers to maintain short-term contracts with her suppliers, i.e., she commits to a supplier for only one period, and selects the winning supplier in each period via a Vickrey auction. The buyer uses information gathered in the first auction to set a reserve price at $T = 2$; if $B^1_{SI}(t^1)$ is the winning bid at $T = 1$, the buyer sets the reserve price at $T = 2$, r^2 to be the incumbent's highest possible cost at $T = 2$, i.e., $r^2 = C + 1 - \alpha$ under NLI and $r^2 = t^1$ under LI[2].

Note that under NLI, the incumbent's highest cost at $T = 2$ is the same regardless of the type of winning supplier at $T = 1$ $(= C + 1 - \alpha)$. Thus, the reserve price is deterministic, i.e., a supplier's strategic behavior at $T = 1$ is irrelevant to buyer's choice of the reserve price. However, under LI, the reserve price is based on the auction at $T = 1$ (i.e., different reserve price according to different type of winning supplier at $T = 1$). Under such a setting, the supplier may be inclined to affect the buyer's choice of the reserve price strategically when he submits his bid at $T = 1$, from which he may take advantage of some information rent. Or, if the buyer is able to audit the supplier's cost after the production and becomes known his actual cost, the supplier's bid at $T = 1$ may not affect the buyer's selection of the reserve price. In this thesis, we consider both cases and compare the differences :

LI1 : the buyer sets the reserve price based on the winning bid $B^1_{SI}(t^1)$.

LI2 : the buyer sets the reserve price based on the incumbent's actual cost at $T = 1$, t^1.

It is common in SI auctions for the buyer to announce upfront that she will not switch suppliers unless the cost savings from an alternative supplier are greater than the cost of switching. Under SI, the buyer adds on the switching cost s to each *submitted* bid by an entrant at $T = 2$ to derive an *effective* bid for each entrant (note that the *submitted* bid and *effective* bid are the same for the incumbent supplier). If, given the effective bids, the lowest cost bidder is an entrant, then the buyer will pay him the second lowest *effective* bid minus s; if an incumbent is selected then she pays him an amount equal to min[reserve

[2]As we will mention later, we find the strictly monotonically increasing $B^1(t^1)$, and hence the buyer is able to infer the bidder type t^1 from his bid.

price, the second lowest *effective* bid] [3]. We define F_E to be the entrants' 'adjusted' cost distribution with support $[C + s, C + 1 + s]$, i.e., $F_E(C + s) = 0, F_E(C + 1_s) = 1$.

Table 2.1: Strategy space in each procurement mechanism.

Mechanism	Buyer	Suppliers
EPC	$R(t^1)$	$B^1_{EPC}(t^1),\ \ B^2_{EPC}(t^2)$
SI		$B^1_{SI}(t^1),\ \ B^2_{SI}(t^2)$

Table 2.1 summarizes the strategy space in each of the two mechanisms. We use $B^T_M(t^T)$ to denote the bidding strategy used by supplier t under procurement mechanism M in the T^{th} auction. Given the frequent use of these two mechanisms, we pose and answer the following two questions:

- What is the optimal eroding price schedule $R(t^1)$ under EPC?

- Can an eroding price contract outperform standard auctions, by yielding the buyer a lower expected total procurement cost?

In this thesis, we consider three different models (NLI, LI1 and LI2) according to the market environments where the buyer would face. Table 2.2 and figure 2.3 show these different market environments in each model (explained in further detail in each corresponding chapter; NLI is discussed in Chapter 3, LI1 and LI2 are discussed in Chapter 4 and 5, respectively).

Besides the comparisons between EPC and SI under various model settings, we observe the optimal reserve prices under SI (in NLI market setting) in chapter 6 (see figure 2.3). We will explain the details of the optimal reserve prices under SI when we go to the chapter 6.

[3] For example, if $s = 2$, the incumbent's bid at $T = 2$ is equal to 10, and the first and second lowest submitted bids from entrants are 6 and 9 respectively, then an entrant wins and is paid 8 (=10 - 2). Note that this payment rule can result in the buyer paying more than the second lowest submitted bid to a winning incumbent at $T = 2$. For example, if $s = 2$, the reserve price at $T = 2$ is 10, the lowest submitted bid from entrants is 9 and the incumbent's bid is 7; then the incumbent will win and be paid 10 (although the second lowest submitted bid is 9, the effective bid is 11). We adopt this payment rule for the sake of consistency.

Table 2.2: Different Settings under NLI, LI1, and LI2

Model Assumptions	NLI (chapter 3)	LI1 (chapter 4)	LI2 (chapter 5)
Suppliers' costs at $T = 2$	No-Lock-in	Lock-in (incumbent only)	
Incumbent's cost t_I^2	$[C - \alpha,\ C + 1 - \alpha]$	$[t^1 - \alpha,\ t^1]^\S$	
Entrant's cost t_E^2	$[C,\ C + 1]$	$[C,\ C + 1]$	
Future (Reserve) Price	based on supplier's bid at $T = 1$		based on supplier's actual cost at $T = 1$

§ t^1 : an incumbent's cost at $T = 1$

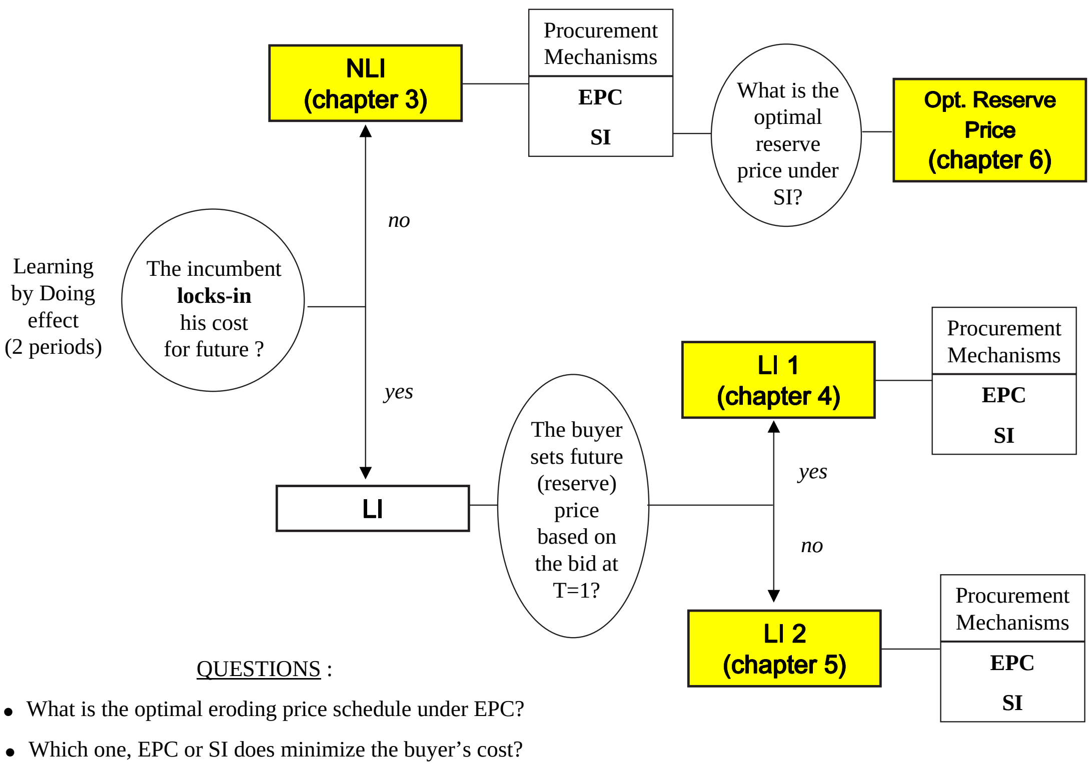


Figure 2.3: Scheme of the procurement mechanisms studied in this thesis

We first consider the NLI setting in next chapter 3. We solve for the suppliers' equilibrium bids and then go on to compare the buyer's expected costs under EPC and SI, respectively. We then perform a similar analysis for the LI1 and LI2 settings in chapter 4 and 5.

2.2 Equilibrium Bidding Strategy at $T = 2$

Before we proceed to analyze the NLI, LI1 and LI2 settings, we make the following observation that is valid under all settings. The 2^{nd} price nature of the auctions simplifies the search for the bidders' optimal bidding behaviors in the last auction: Vickrey[42] demonstrated that it is a (weakly) dominant strategy for the suppliers to bid their true costs in the last auction.

$$B^2_{CE}(t^2) = B^2_{SI}(t^2) = \begin{cases} t^2_I, & \text{if } t \text{ is an incumbent} \\ t^2_E & \text{if } t \text{ is an entrant} \end{cases} \tag{2.1}$$

The assumption of 2^{nd} price auctions allows us to focus our attention on the bidding behavior in the first auction, $B^1_M(t^1)$. Note that the 2^{nd} price nature of the auctions also implies that the bidders do not use any information gathered after the first auction under SI or EPC strategically when submitting a second bid; i.e., the information flow that occurs under SI and EPC does not affect bidding behavior at $T = 2$. In equilibrium, each supplier will adopt a strategy that maximizes his expected profit conditional on what strategies he believes his opponents are using. Given the symmetry in beliefs at $T = 1$, it is natural to posit that two suppliers of the same type will bid the same in equilibrium. Hence, we focus our attention on symmetric equilibrium bidding functions that are continuous and strictly monotonically increasing in type.

CHAPTER III

PROCUREMENT MECHANISMS UNDER NO LOCK-IN SETTING (NLI)

3.1 Model

We begin by first considering a model where the incumbent's baseline cost at $T = 2$ is independent of his cost at $T = 1$.

- **Suppliers' costs at** $T = 2$: every supplier faces a new cost in each period. Thus, incumbent does not lock-in his cost at $T = 2$ same as entrants.
- **Incumbent's cost** t_I^2 **distribution** : the incumbent's cost at $T = 2$ is drawn from the distribution $F_I(t^2) : F(C - \alpha) = 0, F(C + 1 - \alpha) = 1$.
- **Possible value of** $R(t^1)$: $R \in [C - \alpha, C + 1 - \alpha] \ \forall \ t^1$. That is, the buyer does not consider the price that is guaranteed to make the supplier unprofitable at $T = 2$ ($R < C - \alpha$) or pay the supplier more than his possible highest cost ($R > C + 1 - \alpha$)[1].
- **Setting of future price** : reserve price under SI are set to be $C + 1 - \alpha$, i.e. the incumbent's possible highest cost at $T = 2$[2].

Figure 3.1 shows the cost distributions of suppliers in each period.

3.2 Equilibrium Bidding Strategies under EPC and SI

Under NLI setting, any supplier at $T = 1$ faces the same expected profit as an incumbent or an entrant at $T = 2$. As a result, we have our first observation.

OBSERVATION 3.1. *Under NLI, it is optimal for the buyer to offer all suppliers the same price, R, at* $T = 2$*, i.e.,* $R(t^1) = R \ \ \forall t^1$.

[1] Even if we relax the assumption, the optimal solution does not change.
[2] Since α is common knowledge, the timing of the announcement of the reserve price does not affect to any equilibrium strategy of bidders.

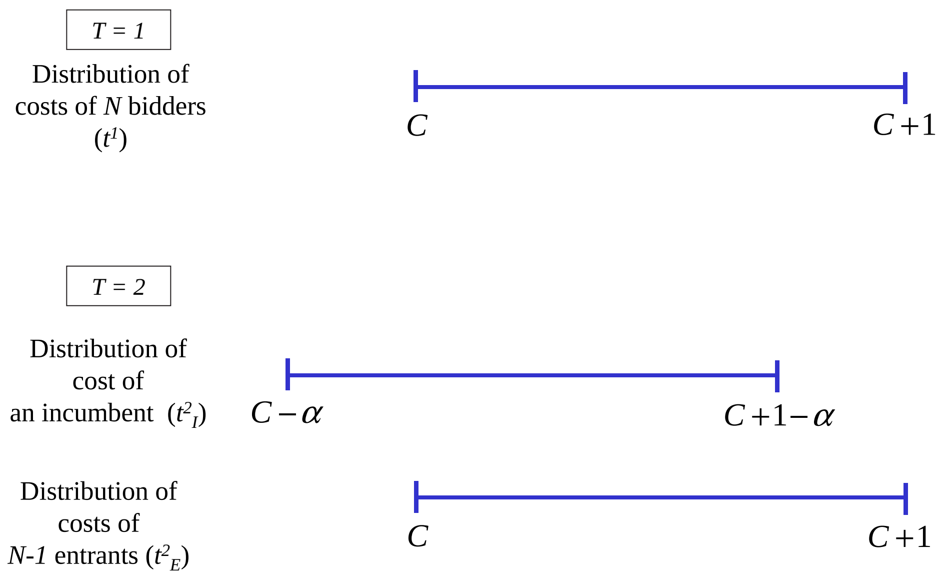


Figure 3.1: Costs at $T = 1$ and $T = 2$ under NLI

To obtain the buyer's expected total costs, we must identify the supplier's equilibrium bidding strategies under each mechanism.

PROPOSITION 3.1. *In combination with the second period bids outlined in equation (2.1), the following constitutes a subgame perfect equilibrium bidding strategy under EPC,*

$$\begin{aligned} B^1_{EPC}(t^1) &= t^1 - \underbrace{\int_{C-\alpha}^{R} F_I(x)\,dx}_{E[\Pi^2_{EPC}(t^2_I);R]} \\ &\quad + \underbrace{[1 - F_I(R)] \int_{C}^{C+1} \left[1 - F_{(1:N-2)}(x)\right] F(x)\,dx}_{E[\Pi^2_{EPC}(t^2_E);R]} \qquad \forall\ t^1 \end{aligned} \tag{3.1}$$

Proof. see appendix A.1. □

We use $F_{(i:j)}$ to denote the distribution of the ith lowest order statistics out of j suppliers. The ability of the incumbent supplier to reject R at $T = 2$ has two opposing effects on a supplier's bid at $T = 1$: (1) the supplier is guaranteed to have a non-negative profit as an incumbent at $T = 2$ and hence the supplier always *shades* his bid by this expected non-negative profit given R ($E[\Pi^2_{EPC}(t^2_I); R]$); and (2) if he loses at $T = 1$, the supplier has the opportunity to bid again with a new set of costs as an entrant (possibly higher or lower costs); this opportunity causes the supplier to *inflate* his bid ($E[\Pi^2_{EPC}(t^2_E); R]$).[3] The fact that all suppliers draw their expected costs at $T = 2$ from the same distributions F_I and F implies that they all shade/inflate their bids by the same amount.

When R or(and) α increases, the supplier's expected profit as an incumbent ($E[\Pi^2_{EPC}(t^2_I); R]$) increases, but his expected profit as an entrant ($E[\Pi^2_{EPC}(t^2_E); R]$) decreases as a result of it being less likely that, should he lose at $T = 1$, an opponent would reject R at $T = 2$. The supplier's profit as an incumbent does not change as N increases, while his profit as an entrant decreases because of the decreasing difference between his bid and the second lowest entrant's bid. This, in turn, exerts a downward pressure on his bid. Note that s does not appear in $B^1_{EPC}(t^1)$; if an auction takes place at $T = 2$, it is only *after* the incumbent has rejected R and involves only entrants.[4]

PROPOSITION 3.2. *In combination with the second period bids outlined in equations (2.1), the following constitutes a subgame perfect equilibrium bidding strategy under SI,*

$$B^1_{SI}(t^1) = \begin{cases} \textit{if } \alpha + s \geq 1, \\ t^1 - [r^2 - E[t^2_I]], \\ \\ \textit{if } \alpha + s < 1, \\ t^1 - \underbrace{\left[\int_{C-\alpha}^{C+s} F_I(x)\,dx + \int_{C+s}^{C+1-\alpha} F_I(x)\left[1 - F_{E(1:N-1)}(x)\right]\left[1 - F_E(x)\right]dx\right]}_{E[\Pi^2_{SI}(t^2_I)]} \\ \quad + \underbrace{\int_{C+s}^{C+1-\alpha} \left[1 - F_I(x)\right]\left[1 - F_{E(1:N-2)}(x)\right] F_E(x)dx}_{E[\Pi^2_{SI}(t^2_E)]}, \end{cases} \tag{3.2}$$

[3] Under the uniform distribution, the supplier t will always submit his bid below his cost since the positive gains as an incumbent at $T = 2$ dominates the gains as an entrant($E[\Pi^2_{EPC}(t^2_I); R] > E[\Pi^2_{EPC}(t^2_E); R]$, see appendix A.3).

[4] As we will see in the next section, the optimal R is increasing in s. Thus, s also indirectly affects the bidding behavior of bidder t.

where $r^2 = C + 1 - \alpha$ *(reserve price)*

Proof. see appendix B.1. $\square$

The incentives for bid shading and inflating are similar under SI and EPC. The opportunity to participate in the second auction with a (possibly) reduced cost after experiencing learning by doing as an incumbent supplier incents a supplier to shade his bid ($E[\Pi^2_{SI}(t^2_I)]$). However, the opportunity to participate with a new cost at $T = 2$ incents a supplier to inflate his bid ($E[\Pi^2_{SI}(t^2_E)]$). The incentive to shade dominates the incentive to inflate, and hence the equilibrium bid is always below t^1 (please see appendix B.3). If $\alpha + s \geq 1$ (i.e., the incumbent's cost at $T = 2$ is always less than or equal to the minimum possible entrant's cost), there is no chance for a supplier to lose at $T = 1$ and then win at $T = 2$, and hence there is no inflation effect on his bid at $T = 1$. As with EPC, all cost types shade their bids by the same amount.

3.3 Expected Total Cost and the Optimal Eroding Price R

Given the equilibrium bids derived in the propositions 3.1 and 3.2, we are one step away from comparing the performance of each procurement mechanism. While the expected costs associated with SI are completely determined by the suppliers' bids, EPC requires the buyer to select an eroding price R (see table 3.1 below). We use $x_{(i:j)}$ to denote the i^{th} lowest order statistic out of j random draws, and P_1 to denote the probability that the incumbent supplier accepts R at $T = 2$, i.e., $P_1 = F_I(R)$ ($P_0 = 1 - P_1$).

Table 3.1: Expected total cost in each procurement mechanism.

Mechanism	Expected Total Cost (ETC)
EPC	$ETC_{EPC}(R) \quad = E[B^1_{EPC}(t^1)_{(2:N)}] + P_1 \times R + P_0 \times E[B^2_{EPC}(t^2)_{(2:N-1)} + s]$
SI	$ETC_{SI} \quad = E[B^1_{SI}(t^1)_{(2:N)}] + E[\min(\tilde{B}^2_{SI}(t^2)^{\S}_{(2:N)}, r^2)]$

$\S\tilde{B}^2_{SI}(t^2)_{(2:N)}$ is the effective bid : if an entrant defines the payment, $\tilde{B}^2_{SI}(t^2)_{(2:N)} = B^2_{SI}(t^2)_{(2:N)} + s$

PROPOSITION 3.3. *Under EPC, the buyer's optimal choice of R^* is as follows.*

$$R^* = \min[\hat{R},\ C+1-\alpha], \tag{3.3}$$

where

$$\hat{R} = \underbrace{\int_C^{C+1} \left[1 - F_{(1:N-2)}(x)\right] F(x)\,dx}_{E[\Pi^2_{EPC}(t^2_E)]} + \underbrace{(C+1+s) - \int_{C+s}^{C+1+s} F_{E(2:N-1)}(x)\,dx}_{E[t^2_{E(2:N-1)}+s]} \tag{3.4}$$

The buyer's expected total cost with R^* *is given by,*

$$ETC_{EPC}(R^*) = \begin{cases} E[t^1_{(2:N)}] + E[t^2_I], & \text{if } R^* = C+1-\alpha \\ \\ E[t^1_{(2:N)}] + E[t^2_I] - \underbrace{\int_{\hat{R}}^{C+1-\alpha} [1 - F_I(x)]\,dx}_{\Psi}, & \text{if } R^* = \hat{R} \end{cases} \tag{3.5}$$

Proof. see appendix A.2.[5] □

R has two opposite effects on the buyer's expected total cost, a decrease in R decreases the buyer's payment at $T = 2$, but increases her payment at $T = 1$ (due to an increase in the bids at $T = 1$, as described in proposition 3.1). The optimal R balances these two effects.[6] It is interesting to note that $\hat{R}$ is *not* a function of α; this is because the factors that influence $\hat{R}$ are the potential profits as an entrant($E[\Pi^2_{EPC}(t^2_E)]$) as well as the expected cost from an auction with only entrants ($E[t^2_{E(2:N-1)} + s]$), neither of which are impacted by α. $\hat{R}$ can be considered the (direct and indirect) cost of the buyer if she runs an additional auction. Note that $E[\Pi^2_{EPC}(t^2_E)]$ and $E[t^2_{E(2:N-1)} + s]$ arise only when the incumbent rejects $\hat{R}$ (with the probability of $1 - F_I(\hat{R})$). That is, the buyer offers to her incumbent the price

[5]The reader should distinguish $E[\Pi^2_{EPC}(t^2_E)]$ from $E[\Pi^2_{EPC}(t^2_E); R]$. The former is the expected profit as an entrant, while the latter is the expected profit conditional on the incumbent rejecting R. Under the uniform distribution, $E[\Pi^2_{EPC}(t^2_E)] = \frac{1}{N(N-1)}$

[6]For example, consider a setting where $R^* = \hat{R} < C + 1 - \alpha$; suppose that the buyer sets $R^* > \hat{R}$, i.e. $R^* = \hat{R} + \epsilon$. The expected cost at $T = 1$ decreases by the amount $\int_{\hat{R}}^{\hat{R}+\epsilon} F_I(x)dx + K \int_{\hat{R}}^{\hat{R}+\epsilon} f_I(x)dx$, where $K = E[t^2_{E(2:N-1)} + s]$, as a result of a lower bids at $T = 1$, but the expected cost at $T = 2$ increases by $\epsilon F_I(\hat{R} + \epsilon) + K \int_{\hat{R}}^{\hat{R}+\epsilon} f_I(x)dx$. Since $F_I(x)$ is a strictly increasing function, the increase in cost at $T = 2$ is larger than the decrease in cost at $T = 1$. Similarly, if the buyer sets $R^* = \hat{R} - \epsilon$, the increasing amount of aggressiveness of bid at $T = 1$ ($\int_{\hat{R}-\epsilon}^{\hat{R}} F_I(x)dx + K \int_{\hat{R}-\epsilon}^{\hat{R}} f_I(x)dx$) becomes larger than the decreasing amount of cost at $T = 2$ ($\epsilon F_I(\hat{R} - \epsilon) + K \int_{\hat{R}-\epsilon}^{\hat{R}} f_I(x)dx$).

which is equal to the cost in case that the incumbent rejects it (not just the direct price from the second auction from entrants, but also the price increased by the second auction by the rejection of the incumbent).[7]

We can make the following observation,

OBSERVATION 3.2. *$R^* = \hat{R}$ is a sufficient condition for $\alpha + s < 1$.*

Proof. From equation (3.4), we can rewrite $\hat{R} = \Gamma + C + s$, where Γ is a positive term (this follows since both integrals in equation (3.4) are positive; furthermore, the second integral represents the expected second lowest order statistic in the range of $[C+s,\ C+s+1]$, which is greater than $C+s$). We have that $\hat{R} = \Gamma + C + s < C + 1 - \alpha \Rightarrow \alpha + s < 1 - \Gamma < 1$. $\square$

Given the optimal R^*, the buyer's expected total cost is composed of her payment to the supplier for the first unit of the good ($E[t^1_{(2:N)}]$) and the cost for the second unit ($E[t^2_I] - \Psi$). The last term $-\Psi = \int_{R^*}^{C+1-\alpha} (R^* - x)\, f_I(x)dx$, is a negative term and can be interpreted as the expected reduction in procurement cost when the incumbent rejects the R^* at $T = 2$ and an entrant supplies the buyer. When the potential entrant pool is relatively uncompetitive (this adjective describes a market when N is small, and/or α and s are large), the buyer does not wish to expose herself to a second auction. Hence she sets $R^* = C + 1 - \alpha$ and keeps the same supplier for both periods, and $\Psi = 0$. However, as the entrant pool becomes competitive (N increases, and/or α and s decrease), the buyer will find it optimal to reduce R below $C + 1 - \alpha$ and reduce her costs by Ψ.

PROPOSITION 3.4. *Given the bidding strategy as in proposition 3.2, the expected total cost of the buyer under SI is as follows.*

[7] When $\hat{R} < C + 1 - \alpha$, the buyer can lower her cost at $T = 2$ even if the incumbent rejects $\hat{R}$ at $T = 2$, i.e., the cost of the second auction $E[t^2_{E(2:N-1)} + s]$ is less than $\hat{R}$.

$$
ETC_{SI} = \begin{cases} E[t^1_{(2:N)}] + E[t^2_I], & \text{if } \alpha + s \geq 1 \\ \\ E[t^1_{(2:N)}] + E[t^2_I] \\ -\underbrace{\int_{C+s}^{C+1-\alpha} [1 - F_I(x)]\, F_{E(2:N-1)}(x)dx}_{\Phi 1} & \text{if } \alpha + s < 1 \\ +\underbrace{\int_{C+s}^{C+1-\alpha} [1 - F_I(x)] \left[1 - F_{E(1:N-2)}(x)\right] F_E(x)dx}_{\Phi 2\, (=E[\Pi^2_{SI}(t^2_E)])} \end{cases} \tag{3.6}
$$

Proof. see appendix B.2. □

In contrast to EPC, the buyer always conducts a second auction with all N suppliers at $T = 2$ under SI. As described for EPC, the chance to participate in the auction at $T = 2$ incents a supplier to inflate his bid, and this exactly translates into the cost of the buyer ($\Phi 2 = E[\Pi^2_{SI}(t^2_E)]$). However, the second auction with N bidders offers the buyer a chance to procure from a more competitive entrant at $T = 2$ without sacrificing a reduction in the number of bidders, which is captured in $\Phi 1$.[8]

Given the buyer's expected total cost under the two mechanisms, we are now ready to compare the performances of the two mechanisms.

PROPOSITION 3.5. *When $\alpha + s \geq 1$, the buyer's expected total costs are the same under EPC and SI.*

Proof. We prove that $\alpha + s \geq 1$ is a sufficient condition of $R^* = C + 1 - \alpha$. Recall that we can rewrite $\hat{R}$ as $\hat{R} = \Gamma + C + s$, where Γ is a positive term. If $\alpha + s \geq 1$, $\Gamma + C + s \geq \Gamma + C + 1 - \alpha > C + 1 - \alpha$. Thus, $R^* = C + 1 - \alpha$ from equation (3.3). □

When $\alpha + s > 1$, the cost ranges of an incumbent and the entrants do not overlap. Under EPC, the optimal $R^* = C + 1 - \alpha$, while the winning supplier at $T = 1$ will always win at $T = 2$ under SI with the reserve price $r^2 = C + 1 - \alpha$. Hence, the two mechanisms

[8] ETC_{SI} (if $\alpha + s < 1$) in equation (3.6) mirrors Grimm(2003)'s findings. She denotes $\Phi 1$ as the "value of competition" and $\Phi 2$ as the "looser's option value".

Table 3.2: Conditions for comparisons of the buyer's expected total costs under EPC and SI

$R^* = C + 1 - \alpha$	$R^* = C + 1 - \alpha$	$R^* = \hat{R}$
α, s large ($\alpha + s \geq 1$)	N small, α, s large($\alpha + s < 1$)	N large, α, s small ($\alpha + s < 1$)
$ETC_{EPC} = ETC_{SI}$	$ETC_{EPC} < ETC_{SI}$ ($\Longleftrightarrow 0 > \Phi 1 - \Phi 2$)	$ETC_{EPC} < ETC_{SI}$ ($\Longleftrightarrow \Psi > \Phi 1 - \Phi 2$)
	$ETC_{EPC} > ETC_{SI}$ ($\Longleftrightarrow 0 < \Phi 1 - \Phi 2$)	$ETC_{EPC} > ETC_{SI}$ ($\Longleftrightarrow \Psi < \Phi 1 - \Phi 2$)

are cost equivalent. However, when the bidders' costs overlap (i.e., $\alpha + s < 1$), either EPC or SI can minimize the buyer's expected total cost. Propositions 3.6 and 3.7 and table 3.2 present the conditions under which each mechanism dominates, while table 3.3 presents

Table 3.3: Numerical Examples of table 3 : except the case of $\alpha + s \geq 1$ (Uniform)

$R^* = C + 1 - \alpha$					$R^* = \hat{R}$					
Market (N, α, s)	$\frac{ETC_{EPC} - ETC_{SI}}{ETC_{SI}}$(%)	$\Phi 1$	$\Phi 2$	R^*	Market (N, α, s)	$\frac{ETC_{EPC} - ETC_{SI}}{ETC_{SI}}$(%)	$\Phi 1$	$\Phi 2$	Ψ	R^*
(3, 0.4, 0.3)	-0.1	0.0007	0.0038	1.6	(8, 0.4, 0.3)	0.2	0.0078	0.0019	0.0005	1.57
(6, 0.4, 0.3)	0.1	0.0047	0.0024	1.6	(20, 0.4, 0.3)	0.1	0.0221	0.0005	0.0195	1.40
(3, 0.1, 0.1)	-0.6	0.0341	0.0512	1.9	(3, 0.03, 0.1)	-0.5	0.0477	0.0620	0.0007	1.93
(3, 0.4, 0.1)	-0.4	0.0052	0.0156	1.6	(3, 0.05, 0.1)	-0.5	0.0435	0.0589	0.0001	1.93
(4, 0.3, 0.2)	0.0	0.0125	0.0120	1.7	(4, 0.3, 0.05)	0.3	0.0330	0.0218	0.0022	1.63
(4, 0.3, 0.4)	-0.1	0.0018	0.0033	1.7	(4, 0.3, 0.1)	0.2	0.0246	0.0183	0.0001	1.68

representative numerical instances that support each scenario.

PROPOSITION 3.6. *When $R^* = C + 1 - \alpha$ and $\alpha + s < 1$, the buyer's expected total cost under EPC is less than that under SI when $\Phi 1 < \Phi 2$ is satisfied. Conversely, SI outperforms EPC when $\Phi 1 > \Phi 2$ is satisfied.*

PROPOSITION 3.7. *When $R^* = \hat{R}$, the buyer's expected total cost under EPC is less than that under SI when $\Psi > \Phi 1 - \Phi 2$ is satisfied. Conversely, SI outperforms EPC when $\Psi < \Phi 1 - \Phi 2$ is satisfied.*

The buyer will generally find it optimal to set $R^* = C + 1 - \alpha$ when the entrant pool is uncompetitive, i.e., N is small and/or when α or s are large. Whether or not SI will outperform EPC then depends on which factor exerts the larger force; the expected cost savings from holding an auction at $T = 2$ with N bidders ($\Phi 1$) versus the degree to which bidders inflate their bid at $T = 1$ due to the presence of the second auction($\Phi 2$). As the entrant pool becomes more competitive (N increases and/or α or s decrease), the buyer will find it optimal to set $R^* = \hat{R} < C + 1 - \alpha$ and expose herself to the incumbent rejecting the contract at $T = 2$ and running a second auction with an associated cost savings of Ψ. Whether or not EPC dominates SI will depend on the value of having an addition bidder and having the price be competitively determined at $T = 2$.

While the derivation of the equilibrium bids and optimal R^* proved to be an interesting theoretical exercise, we found that the actual cost difference between EPC and SI was very small in various distributions, e.g. when F follows beta distributions with different beta functions (e.g. $B(1, 1)$ - uniform, $B(2, 3)$, $B(2, 2)$, $B(3, 2)$). From our numerical examples, as a general rule, we found that SI will dominate EPC if $R^* = \hat{R} < C + 1 - \alpha$; the only exception to the rule occurred when $N = 3$ (see table 3.4, 3.5, 3.6, and 3.7).

OBSERVATION 3.3. *The expected cost difference between EPC and SI is less than* 1%*.*

Table 3.4: Comparison of EPC and SI under NLI : Beta $B(1,1)$ - Uniform

$s = 0.1$						
$\alpha \diagdown N$	3	4	5	6	10	20
0.1	-0.6 (t_H)	0.5 ($\hat{R}$)	0.6 ($\hat{R}$)	0.5 ($\hat{R}$)	0.3 ($\hat{R}$)	0.1 ($\hat{R}$)
0.2	-0.6 (t_H)	0.4 ($\hat{R}$)	0.6 ($\hat{R}$)	0.5 ($\hat{R}$)	0.3 ($\hat{R}$)	0.1 ($\hat{R}$)
0.3	-0.5 (t_H)	0.2 ($\hat{R}$)	0.5 ($\hat{R}$)	0.5 ($\hat{R}$)	0.3 ($\hat{R}$)	0.1 ($\hat{R}$)
0.4	-0.4 (t_H)	0.0 (t_H)	0.4 ($\hat{R}$)	0.5 ($\hat{R}$)	0.3 ($\hat{R}$)	0.1 ($\hat{R}$)
0.5	-0.3 (t_H)	-0.1 (t_H)	0.1 (t_H)	0.3 ($\hat{R}$)	0.3 ($\hat{R}$)	0.1 ($\hat{R}$)
0.7	0.0 (t_H)	0.0 (t_H)	0.0 (t_H)	0.0 (t_H)	0.1 (t_H)	0.1 ($\hat{R}$)
$\alpha = 0.4$						
$s \diagdown N$	3	4	5	6	10	20
0.1	-0.4 (t_H)	0.0 (t_H)	0.4 ($\hat{R}$)	0.5 ($\hat{R}$)	0.3 ($\hat{R}$)	0.1 ($\hat{R}$)
0.2	-0.2 (t_H)	-0.1 (t_H)	0.1 (t_H)	0.3 ($\hat{R}$)	0.3 ($\hat{R}$)	0.1 ($\hat{R}$)
0.3	-0.1 (t_H)	-0.1 (t_H)	0.0 (t_H)	0.1 (t_H)	0.2 ($\hat{R}$)	0.1 ($\hat{R}$)
0.4	-0.0 (t_H)	-0.0 (t_H)	-0.0 (t_H)	0.0 (t_H)	0.1 (t_H)	0.1 ($\hat{R}$)
0.5	-0.0 (t_H)	-0.0 (t_H)	0.0 (t_H)	0.0 (t_H)	0.0 (t_H)	0.0 (t_H)
0.7	0.0 (t_H)	0.0 (t_H)	0.0 (t_H)	0.0 (t_H)	0.0 (t_H)	0.0 (t_H)

⋇ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\ \%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.
⋇⋇ The optimal R^* is shown in the parenthesis (where $t_H \equiv C + 1 - \alpha$).

Table 3.5: Comparison of EPC and SI under NLI : Beta $B(2,2)$

$s = 0.1$							$\alpha = 0.4$						
$\alpha \diagdown N$	3	4	5	6	10	20	$s \diagdown N$	3	4	5	6	10	20
0.1	-0.5	0.2	0.3	0.3	0.3	0.1	0.1	-0.2	-0.1	0.0	0.1	0.2	0.1
0.2	-0.4	0.1	0.2	0.3	0.3	0.1	0.2	-0.1	0.0	0.0	0.0	0.1	0.1
0.3	-0.3	0.0	0.1	0.2	0.2	0.1	0.3	0.0	0.0	0.0	0.0	0.0	0.1
0.4	-0.2	-0.1	0.0	0.1	0.2	0.1	0.4	0.0	0.0	0.0	0.0	0.0	0.0
0.5	-0.1	0.0	0.0	0.0	0.1	0.1	0.5	0.0	0.0	0.0	0.0	0.0	0.0

※ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.

Table 3.6: Comparison of EPC and SI under NLI : Beta $B(2,3)$

$s = 0.1$							$\alpha = 0.4$						
$\alpha \diagdown N$	3	4	5	6	10	20	$s \diagdown N$	3	4	5	6	10	20
0.1	-0.3	0.1	0.2	0.3	0.2	0.1	0.1	-0.1	0.0	0.0	0.1	0.1	0.1
0.2	-0.3	0.1	0.2	0.2	0.2	0.1	0.2	0.0	0.0	0.0	0.0	0.0	0.0
0.3	-0.2	0.0	0.1	0.1	0.1	0.1	0.3	0.0	0.0	0.0	0.0	0.0	0.0
0.4	-0.1	0.0	0.0	0.1	0.1	0.1	0.4	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0

※ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.

Table 3.7: Comparison of EPC and SI under NLI : Beta $B(3,2)$

$s = 0.1$							$\alpha = 0.4$						
$\alpha \diagdown N$	3	4	5	6	10	20	$s \diagdown N$	3	4	5	6	10	20
0.1	-0.5	0.0	0.2	0.3	0.3	0.2	0.1	-0.1	-0.1	0.0	0.0	0.1	0.1
0.2	-0.4	-0.1	0.1	0.2	0.3	0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0
0.3	-0.2	-0.1	0.0	0.1	0.2	0.1	0.3	0.0	0.0	0.0	0.0	0.0	0.0
0.4	-0.1	-0.1	0.0	0.0	0.1	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0

※ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.

CHAPTER IV

PROCUREMENT MECHANISMS UNDER LOCK-IN SETTING 1 (LI1)

4.1 Model

In previous chapter (NLI), we study the procurement mechanisms where all suppliers redraw their cost at $T = 2$ from the same distribution $\sim [C - \alpha, C + 1 - \alpha]$, if they wins at $T = 1$. By this common cost distribution for any type of an incumbent, (1) all supplier types shade/infalte their bids by same amount, and (2) the buyer is indifferent from the type of the winner at $T = 1$, and hence she announces a *single* value R as a eroding price at $T = 2$ under EPC.

Different from NLI, in some market settings, an incumbent's cost at $T = 2$ depends on his cost at $T = 1$. For example, the incumbent supplier may be able to write a long-term contract fixing his own input prices over both periods. Under such settings, the incumbent's cost at $T = 2$ is drawn from the range of values $[t^1 - \alpha,\, t^1]$.

Since the cost distribution of an incumbent depends on his type at $T = 1$, the dynamics of the bidder's behaviors as well as the choice of the optimal $R(t^1)$ under a LI1 setting become more complex when compared to those under NLI. For example, when the bidder submits his bid at $T = 1$, he must consider the winning opponent's cost $T = 1$ should he lose and its impact on his potential profits at $T = 2$. The same is not true under NLI.

Similarly, the buyer's selection of the optimal eroding price contract is complicated by the incumbent's temporal cost dependency; it is no longer optimal to offer all supplier types the same price at $T = 2$. Rather, the buyer should offer a *schedule* of R values, $R(t^1)$, one for each type of supplier at $T = 1$. (Note: The buyer is able to infer a supplier's type by his bid at $T = 1$ under strictly monotonically increasing bid functions. Hence, we can model the eroding price schedule as a function of a supplier's true type, i.e., $R(B^1_{EPC}(t^1)) =$

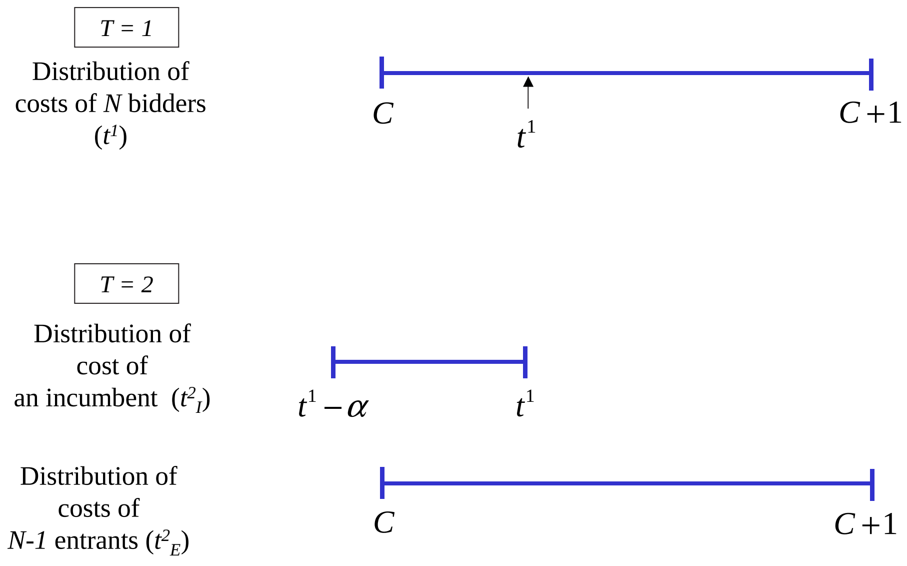


Figure 4.1: Costs at $T = 1$ and $T = 2$ under LI1

$R(t^1)$). In our discussions with companies that have employed EPC[1], we learned that *non-discriminatory* eroding price contracts are the most commonly used format, i.e., *any* winning supplier is required to reduce its price in the second period by the same fixed percent $(1 - k)$ and is paid the price $R(t^1) = kt^1$ where $0 < k \leq 1$. Markets for which non-discriminatory eroding price contracts have been used in FreeMarkets auctions include PCA/PCBs, capacitors, transceivers, monitors, and wire harnesses, please see table1.1 in chapter 1 for additional examples.

We summarize LI1 model settings described above as follows (the basic scheme of the model is described in chapter 2).

- **Suppliers' costs at** $T = 2$: The supplier who wins the auction at $T = 1$ 'locks-in' his (baseline) cost for the future; all other suppliers redraw their costs at $T = 2$.

[1]During the summer of 2003, Se-kyoung Oh had the benefit of interning at FreeMarkets.com, where she was able to observe their auction practices across multiple markets. We wish to extend our thanks to Jack Allamon, Bill Blair, Michael Bryson, Brenna Bulwinkle, and David Talbot for sharing with us their time and auction experiences.

- **Incumbent's cost t_I^2 distribution** : the incumbent's cost at $T = 2$ is drawn from the distrtibution $F_I(t_I^2)$: $F(t^1 - \alpha) = 0,\ F(t^1) = 1$.

- **Possible value of $R(t^1)$** : $R(t^1) \in [t^1 - \alpha, t^1]\ \forall\ t$. That is that the buyer does not consider price schedules that are guaranteed to make the supplier unprofitable at $T = 2$ ($R(t^1) < t^1 - \alpha$) or pay the supplier more than his first period cost ($R(t^1) > t^1$).[2]

- **Setting of future price** : $R(t^1)$ and reserve price under SI are based on the supplier's bid at $T = 1$. That is, the payment to the incumbent will be influenced by the supplier's bid at $T = 1$.

Figure 4.1 shows the cost distributions of suppliers in each period (refer figure 3.1 for comparison of NLI and LI1).

4.2 Equilibrium Bidding Strategy and the Optimal Eroding Price $R(t^1)$ under EPC

We first solve for the equilibrium bidding strategies under EPC.

PROPOSITION 4.1. *In combination with the second period bids outlined in equation (2.1), the following constitutes a subgame perfect equilibrium bidding strategy under EPC,*

$$\begin{aligned} B^1_{EPC}(t^1) &= t^1 - \underbrace{\int_{t^1-\alpha}^{R(t^1)} F_I(x)\,dx}_{E[\Pi^2_{EPC}(t^2_I);R(t^1)]} + \underbrace{\left[1 - F_I(R(t^1))\right] \int_{C}^{C+1} \left[1 - F_{(1:N-2)}(x)\right] F(x)\,dx}_{E[\Pi^2_{EPC}(t^2_E);R(t^1)]} \\ &\quad + R'(t^1) F_I(R(t^1)) \left(\frac{1 - F(t^1)}{(N-1) f(t^1)} \right), \qquad \forall\ t^1 \qquad (4.1) \end{aligned}$$

Proof. see appendix C.1 □

In contrast with $B^1_{EPC}(t^1)$ under NLI (equation (3.1)), suppliers may inflate or shade their bids at $T = 1$ differentially (i.e., not by the same amount). In equilibrium, supplier t submits a bid that reflects his unit cost at $T = 1 (t^1)$, shaded by his expected profit at $T = 2$ as an incumbent ($E[\Pi^2_{EPC}(t^2_I); R(t^1)]$). However, this downward pressure on a supplier's

[2]Thus, we will only consider $\max\{1 - \frac{\alpha}{t^1} : t \in [C, C+1]\} \leq k \leq 1$.

bid is countered by (1) the likelihood that he is the lowest cost supplier in the auction at $T = 1$ ($\frac{1-F(t^1)}{(N-1)f(t^1)}$), and the attractiveness of misrepresenting his type at $T = 1$ in order to affect his payment at $T = 2$ ($R'(t^1) = \frac{\partial R(t^1)}{\partial t^1}$), and (2) the possibility of earning money in the second auction, if it is held ($E[\Pi^2_{EPC}(t^2_E); R(t^1)]$).

The eroding price contract influences a supplier's bid in two ways. The *absolute value* of $R(t^1)$ directly affects a supplier's expected profitability at $T = 2$. The higher a supplier's expected profit at $T = 2$, the more aggressively he will bid at $T = 1$. Conversely, if $R(t^1)$ is set so low as to make profitable production at $T = 2$ very unlikely, then a supplier will compensate for this future negative profit stream by inflating his bid at $T = 1$ proportionately. The *shape* of $R(t^1)$ affects how competitively a supplier bids at $T = 1$ with respect to his competitors. As $R'(t^1)$ increases, supplier t has a greater incentive to bid as a higher type and hence secure a higher payment at $T = 2$, if he should win at $T = 1$.

As we saw in proposition 3.1 in section 3.2 , the ability of the incumbent supplier to reject $R(t^1)$ at $T = 2$ exerts both bid shading and bid inflating forces on his bid at $T = 1$[3].

Given the increased complexity of the buyer's decision problem, i.e., determining an entire schedule of prices as opposed to a single price, we now limit our focus to a setting where when F follows the uniform distribution. While this assumption was not needed to derive the equilibrium bidding strategies, it was necessary in order to derive the optimal eroding price contract (we were unable to solve the buyer's decision problem under the general distribution).[4]

PROPOSITION 4.2. *Under the non-discriminatory eroding price schedule $R(t^1) = kt^1$, the buyer's expected total cost under EPC is minimized when $k = 1$. The buyer's expected total cost is as follows.*

$$ETC_{EPC}(R^*(t^1)) = E[t^1_{(2:N)}] + E[t^2_{I(2:N)}] \tag{4.2}$$

[3]A bidder t's expected profit as an entrant is considered only if an incumbent $w(\neq t)$ rejects the buyer's offer $R(w^1)$ at $T = 2$. This is captured in $1 - F_I(R(t^1))$ in the last term in equation (4.1). At $T = 1$, the incumbent's cost w^1 should be no greater than t^1 to be the winner ($w^1 \sim [C, t^1]$, and hence $1 - F_I(R(t^1))$ is the probability that the incumbent with the marginal cost, t^1 rejects $R(t^1)$.

[4]To get an idea of the robustness of our result, we used numerical methods to solve for the optimal k under different beta distributions (e.g. beta functions $B(2, 3)$, $B(2, 2)$, $B(3, 2)$). We found that in all settings, the optimal solution was always $k^* = 1$. This leads us to conjecture that $k^* = 1$ under most general distribution settings.

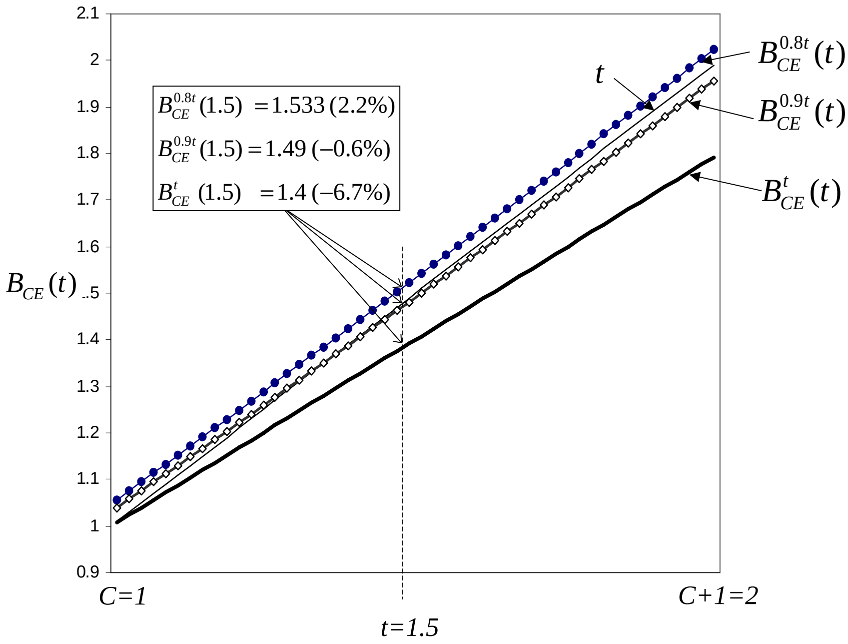


Figure 4.2: Equilibrium bidding strategies in non-discriminatory EPC, with $N = 6$, $\alpha = 0.4$ and $s = 0.1$

Proof. see appendix C.2 and C.4. $\square$

One might initially think that the buyer would find it optimal to set $k < 1$, due to the possibility of a high type supplier winning at $T = 1$. With $k < 1$, the buyer could then hopefully procure from a lower cost supplier in the auction at $T = 2$ if the incumbent does not experience a sufficient cost reduction. However, a non-discriminatory $R(t^1)$ does not offer the buyer the opportunity to differentially set k for each t^1. That, in combination with the fact that suppliers' bids at $T = 1$ become more aggressive as k increases and the incumbent's cost at $T = 2$ is bounded from above by t^1 (which was not the case under NLI) implies that it is optimal for the buyer to guarantee a non-negative profit stream to any incumbent supplier at $T = 2$, and hence keep her incumbent supplier with probability equal to one.

It is illuminating to observe the shifts in $B_{EPC}^{kt}(t)$ as k increases. Figure 4.2 illustrates that suppliers bid more aggressively as k increases: This reflects the increase in expected profit at $T = 2$ if the supplier should win at $T = 1$. For example, under $k = 0.8$, the supplier with cost $t = 1.5$ rejects $R(t)$ with a probability of 75%; he takes this into account when submitting his bid at $T = 1$ and inflates his bid above cost (2.2% inflation) to reflect the opportunity cost of winning at $T = 1$ and not being able to participate in an auction at $T = 2$ with a new and potentially lower cost. As k increases, the probability that the incumbent rejects the contract decreases significantly ($Pr(\text{reject}) = 37.5\%$ for $k = 0.9$, is equal to 0 for $k = 1$). When $k = 1$, supplier $t = 1.5$ *shades* his bid below his cost by 6.7%. Again, the differential bid shading reflects the suppliers' relative competitiveness, as well as their expected profits at $T = 2$, should they win at $T = 1$.

The rate of decrease in bids in combination with the reduced likelihood of incurring s dominates the rate of the increase in $R(t^1)$ (the buyer's expected total cost is decreasing convex function of k - appendix C.2), and hence $k = 1$ minimizes the buyer's total expected costs[5].

4.3 Equilibrium Bidding Strategy under SI and Comparisons between EPC and SI

While we were able to derive a closed form solution for the bids under EPC, we were unable to do so under SI [6].

Table 4.1: Parameters used for $B^1_{SI}(t)$

C	1 , $t \sim U[1, 2]$
N	3, 4, 5, ..., 10, 20, 30
α	0.05, 0.1, 0.2, ..., 0.9
s	0.05, 0.1, 0.2, ..., 0.9

[5]In appendix C.3, we show the sensitivity of the choice of $R^*(t^1) = kt^1$. The buyer's cost are more sensitive to a chance of k (i.e., his cost would rapidly increase), as α and N decrease and s increases.

[6]The bid monotonicity assumption was violated under a few market settings; this implies that strictly monotonically increasing bids may fail to exist for SI when F is continuous. Our next step was to assume a discrete distribution for the supplier type space, and use numerical methods to solve for the equilibrium bids. Under this altered setting, we were able to find equilibrium bids that satisfy strict monotonicity. For each supplier type, we iteratively search for the optimal bid which maximizes his profit given other suppliers' bids, $\Pi_i(B_1, ..., B_{i-1}, B_{i+1}, ..., B_n)$ (see appendix D for the payoff function of bidder t). MATLAB was used for this experiments (interested readers may contact the authors for a copy of the algorithm)

We ran our algorithm over the experimental settings in table 4.1 and varied the values of α, s, and N to capture a wide spectrum of market settings (based on this, we obtained the buyer's expected total cost under SI and compared it with the buyer's cost under EPC. We ran a total of 1210 market instances.)[7]. We approximate the uniform distribution of supplier types by discretizing the uniform distribution over the interval $[C, C+1]$ by increments of $\frac{1}{n}$, i.e., $(t_1, t_2, \ldots, t_n) = (C, C+\frac{1}{n} \ldots, C+\frac{n-1}{n})$. Under these market settings, we first discuss the equilibrium bidding behavior under SI and proceed to compare the buyer's expected total cost under SI and EPC.

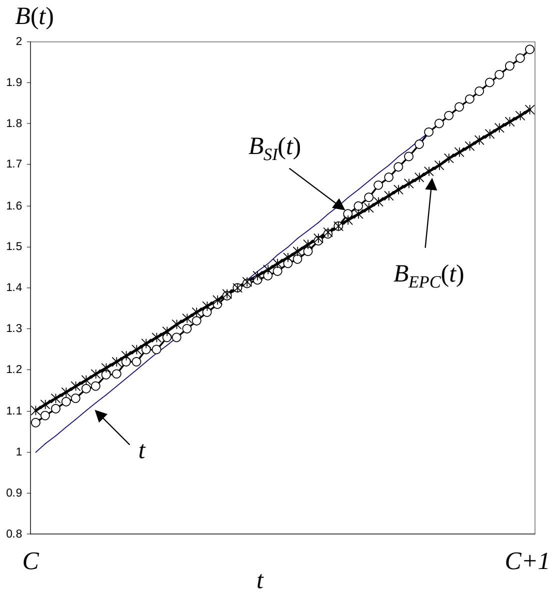
Figure 4.3 plots the equilibrium bids under SI and EPC in four different market settings. As we can see, the bids under SI and EPC are similar for low supplier types, while the difference become larger for high types.

Under EPC, the supplier who wins at $T = 1$ will always supply the buyer at $T = 2$ (and there will be no second auction if he should lose at $T = 1$ and hence there is no opportunity cost associated with winning at $T = 1$). As a result, the bidder submits an aggressive bid to win at $T = 1$; for example, the highest type $(= C + 1)$ shades his bid by amount of his expected profit at $T = 2$, $E[\Pi^2_{EPC}(t^2_I)]$. Under SI, the supplier who wins at $T = 1$ must compete again for the buyer's business at $T = 2$. For a high type, his cost at $T = 2$ (after experiencing a cost reduction) may not be competitive vis-a-vis the entrants' costs, leaving open the possibility of losing at $T = 2$. Conversely, should he lose at $T = 1$, he can reenter the auction at $T = 2$ with a new and potentially more competitive cost, hence his bid at $T = 1$ is conservative. For lower types, it is more likely that a supplier who wins at $T = 1$ will also win in the auction at $T = 2$, and his bid at $T = 1$ reflects this in the form of greater bid shading. For a very low type, they will win again at $T = 2$ with almost certainty and be paid the reserve price (equal to his cost t^1), as would be the case under EPC.

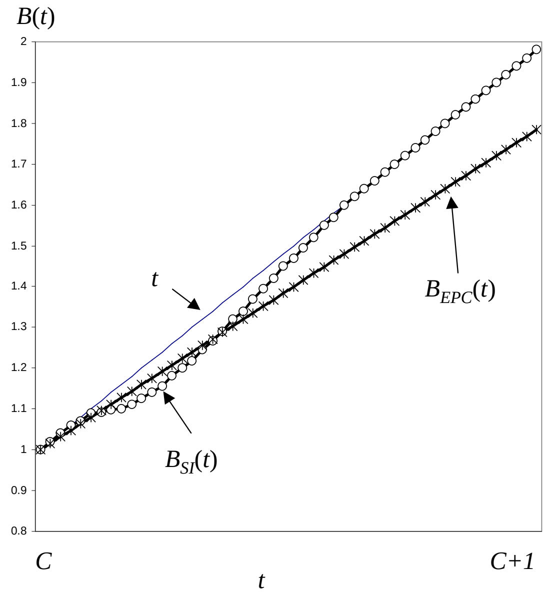
Given the equilibrium bidding strategies, we next compare the buyer's expected total cost under EPC and SI.

OBSERVATION 4.1. *The performance of EPC when compared to SI improves as we*

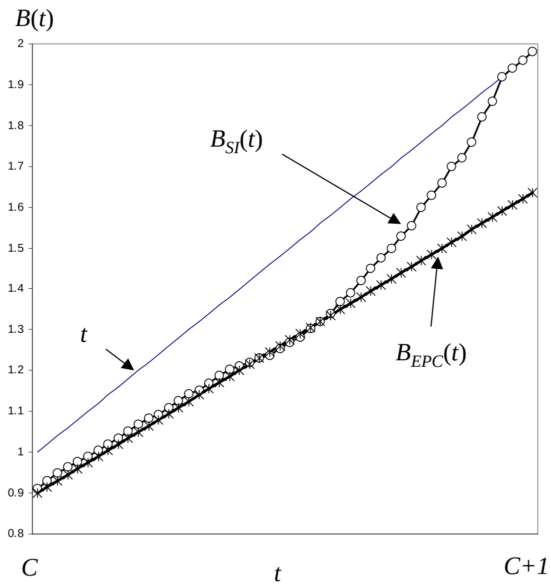
[7]Results illustrated in this section are based on experiments with $n = 50$.



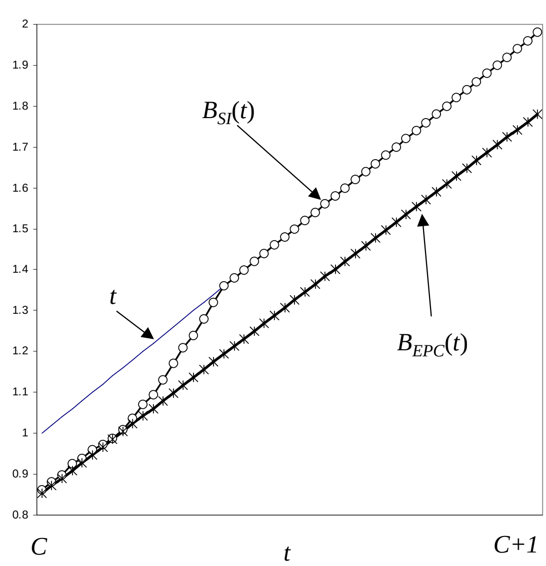
(i) $N = 5$, $\alpha = 0.3$, $s = 0.4$



(ii) $N = 6$, $\alpha = 0.4$, $s = 0.1$



(iii) $N = 5$, $\alpha = 0.7$, $s = 0.4$



(iv) $N = 20$, $\alpha = 0.4$, $s = 0.1$

Figure 4.3: Bids under EPC and SI in various markets

Table 4.2: Comparison of non-discrim. EPC ($k = 1$) and SI under LI1

$s = 0.1$							$\alpha = 0.1$						
$\alpha \diagdown N$	3	4	5	6	10	20	$s \diagdown N$	3	4	5	6	10	20
0.1	2.8	3.4	3.3	3.1	1.7	0.1	0.1	2.8	3.4	3.3	3.1	1.7	0.1
0.2	2.5	2.9	2.8	2.1	0.6	-0.4	0.2	2.3	2.6	2.4	2.2	1.3	-0.1
0.3	2.3	2.3	1.7	1.2	0.0	-0.5	0.3	1.9	2.0	1.8	1.5	1.1	-0.1
0.4	1.9	1.5	1.1	0.6	-0.1	-0.6	0.4	1.5	1.5	1.4	1.2	1.1	-0.1
0.5	1.3	1.0	0.5	0.2	-0.3	-0.8	0.5	1.3	1.3	1.3	1.1	1.1	-0.1
0.7	0.5	0.1	-0.1	-0.2	-0.6	-1.1	0.7	1.2	1.2	1.2	1.1	1.1	-0.1
$s = 0.4$							$\alpha = 0.4$						
$\alpha \diagdown N$	3	4	5	6	10	20	$s \diagdown N$	3	4	5	6	10	20
0.1	1.5	1.5	1.4	1.2	1.1	-0.1	0.1	1.9	1.5	1.1	0.6	-0.1	-0.6
0.2	1.5	1.3	1.0	0.9	-0.1	-0.5	0.2	1.4	1.1	0.5	0.0	-0.4	-0.6
0.3	1.1	0.8	0.6	0.2	-0.5	-0.5	0.3	1.0	0.7	0.1	-0.3	-0.5	-0.6
0.4	0.8	0.5	0.0	-0.4	-0.6	-0.6	0.4	0.8	0.5	0.0	-0.4	-0.6	-0.6
0.5	0.4	0.0	-0.4	-0.5	-0.7	-0.8	0.5	0.6	0.4	-0.1	-0.4	-0.6	-0.6
0.7	-0.2	-0.5	-0.6	-0.7	-0.9	-1.1	0.7	0.6	0.3	-0.1	-0.5	-0.6	-0.6

⁎ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.

increase s, α or N. The buyer's expected total cost under EPC is less than or (at least) same as that under SI when at least two of the market parameters (N, α, and s) are large. Conversely, her cost under SI is less than that under EPC when at least two of the parameters are small.

Table 4.2 illustrates this observation as we vary N, α, and s. Note that the effect of an increase in α is more significant than that of s. For example, when $N = 6$, $\alpha = 0.1$ and $s = 0.1$, the difference is 3.1% (SI outperforms EPC). If s increases to 0.4, it decreases to 1.2%. However, if α increases to 0.4, the difference is further reduced to 0.6%.

As α increases, the incumbent become more competitive at $T = 2$. Under EPC, the incumbent fully reflects this advantage to his bid at $T = 1$, i.e., his bid at $T = 1$ becomes more aggressive by the exact amount of the increased cost reduction at $T = 2$. That is, the fact that the buyer will always keep her incumbent at $T = 2$ ($R^*(t^1) = t^1$) extracts the most aggressive bid from the bidder (e.g., with other settings fixed, if α increases from 0.1 to 0.3, bidder t's bid at $T = 1$ decreases by 0.1 ($= 0.15 - 0.05$), which is the increase of the expected cost reduction). However, it is not the case under SI. Under SI, the incumbent would lose at $T = 2$ with a positive probability. Thus, the increase of α is not fully captured to the bid at $T = 1$. Combined with the fact that the buyer's cost at $T = 2$ under SI also does not fully capture the increase of cost reduction, the rate of the buyer's cost decrease at $T = 1$ under EPC offsets the rate of the decrease at $T = 2$ under SI, hence EPC becomes more attractive to the buyer as α increases.

The change of the switching cost s does not affect both the buyer's expected total cost and the bids under EPC (see equation (4.2) in proposition 4.2). Under SI, the buyer's costs at $T = 1$ and 2 increase when s increases by some degree, and hence EPC becomes attractive to the buyer. Note that with large s, the incumbent would always be the winner and paid the reserve price at $T = 2$, which implies that the cost at $T = 2$ would not change under SI. Therefore, after some degree of s increases, the difference between EPC and SI would remain same.

The change of supplier's cost structures (or buyer's cost), i.e., change of α and s, illustrate that EPC becomes attractive to the buyer as the incumbent becomes competitive. However, the change of N has different effect on the comparisons between the buyer's total costs under EPC and SI.

As we see in table 4.2, with larger N, i.e., entrants become competitive at $T = 2$, EPC becomes attractive to the buyer. When N increases, the cost difference at $T = 2$ between the incumbent and the lowest entrant decreases, which affects the supplier's expected profit at $T = 1$ under SI, while it does not under EPC.

As opposed to the NLI setting, the cost differences can be significant under LI1, ranging anywhere from -1.3% to 5%. This implies that it would be worthwhile for the buyer to educate herself about her supplier pools' cost characteristics (i.e., s and α) as well as the size of the potential bidder pool N before determining whether or not to offer a longer-term relationship to the winner in the first auction. It is worth reiterating that under the optimal

EPC, the buyer does not actually ask to share in any of the cost reductions at $T = 2$ ($k = 1$); therefore the superior performance under EPC is arising from the long-term stability of the relationship.

4.4 Extensions and Managerial Insights

4.4.1 Discriminatory $R(t^1)$ under LI1

Under the NLI setting, the potentially complicated eroding price schedule $R(t^1)$ reduces its form to a simple R due to the independence of the incumbent's cost over time. However, under the LI setting, the buyer should set a different price for each supplier type t^1. In practice, the buyers simplify a potentially complex price schedule by offering mainly nondiscriminatory contracts where all suppliers are asked for the same fractional price reduction, $R(t^1) = kt^1$. While using non-discriminatory price schedule may simplify the buyer's problem, it may also leave some 'money on the table' for the suppliers. If the buyer were to offer a discriminatory price schedule, $R(t^1) = \kappa(t^1)t^1$, it is clear that the buyer would be better off (or at least no worse off). What is not clear is how much better off would she be, what would be the form of the optimal contract and if the cost savings would warranty the added complexity of the contract design.

We assume that $R(t^1)$ is differentiable everywhere and is strictly monotonically increasing in t^1. Thus a buyer would never require a lower price from a higher type t^1; this assumption is particularly reasonable since the suppliers are assumed to have no control on their cost reduction, i.e., the cost reduction arises merely out of having supplied the buyer once before and is not a direct result of any cost-saving measures. By solving a nonlinear program, we were able to find the buyer's optimal $R(t^1)$ (or $\kappa(t^1)$) for a bidder t.

OBSERVATION 4.2. *Under the LI1 setting, the buyer finds in optimal to offer a contract $R(t^1) = \kappa(t^1)t^1$ that has the following properties: (1) guarantee for both 'low' and 'high' cost suppliers that they will not be unprofitable at $T = 2$ (i.e., $\kappa(t^1) \approx 1$) and (2) extract some potential cost reductions for 'moderate' cost suppliers(i.e., $\kappa(t^1) < 1$).*

The buyer finds it optimal to set $\kappa(t^1) = 1$ for high cost types, thereby eliciting the most aggressive bidding behavior from them at $T = 1$. That, combined with the probability of a

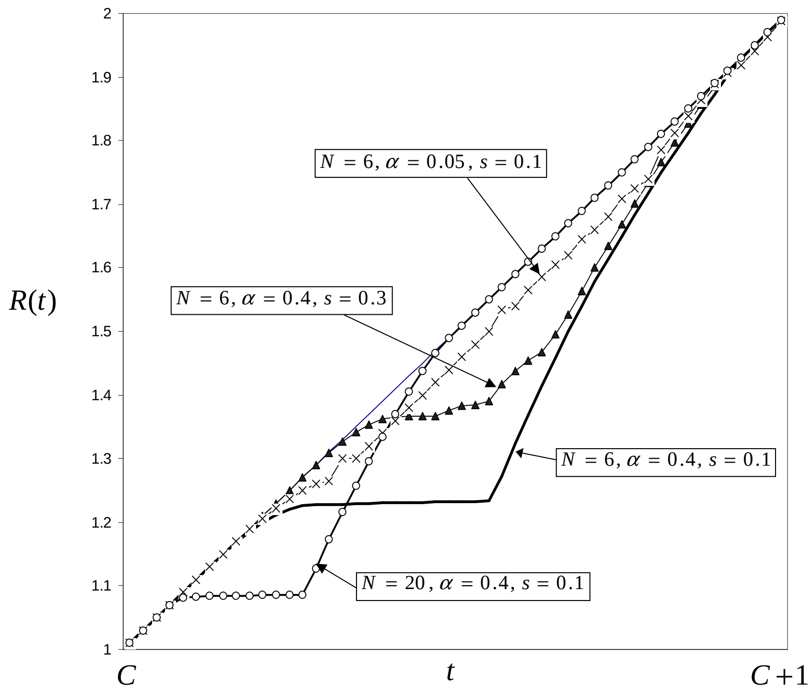


Figure 4.4: Optimal discriminatory $R(t^1)$ for different market settings $(N, \alpha,\ s)$ under LI1

'high' cost supplier winning at $T = 1$ being low, renders $\kappa(t^1) = 1$ optimal. The aggressive bidding behavior from this 'high' cost group cascades down and exerts a downward pressure on 'low' cost suppliers' bids. The reason for $\kappa(t^1) = 1$ for low cost suppliers is the incumbent supplier's ability to reject $R(t^1)$ at $T = 2$. Since (i) a low cost supplier will reject any unprofitable contract, and (ii) the probability that the expected payment at $T = 2$ from the auction plus switching cost is less than the incumbent's unit cost is low (the incumbent's cost at $T = 1$ is the lowest order statistic of N random draws, while the winner at $T = 2$ would be the lowest order statistic of $N - 1$ random draws), the buyer finds it optimal to set $\kappa(t^1) = 1$ for low cost suppliers. For the 'moderate' cost suppliers, the buyer finds it optimal to set $\kappa(t^1) < 1$, in the hopes of securing a lower price at $T = 2$ if the incumbent supplier should win and accept $R(t^1)$ and, if not, of securing a price from the second auction that is comparable to $R(t)$. The degree of $\kappa(t^1) < 1$ and the range of the types differ by the market settings (N, α, s) (see figure 4.4)[8].

[8] As we see in figure 4.4, The definition of the 'moderate' group changes across markets; this group shifts down (towards C) as N increases (N increases from 6 to 20), and shifts up (towards $C + 1$) as s increases

Table 4.3: Comparison of discrim. EPC and SI under LI1

$s = 0.1$							$\alpha = 0.1$						
$\alpha \setminus N$	3	4	5	6	10	20	$s \setminus N$	3	4	5	6	10	20
0.1	2.8	2.7	2.4	2.0	0.7	-0.3	0.1	1.9	1.1	0.4	-0.1	-0.6	-0.8
0.2	2.5	2.1	1.9	0.9	-0.1	-0.7	0.2	1.4	0.9	0.2	-0.4	-0.7	-0.8
0.3	2.3	1.6	0.9	0.4	-0.6	-0.7	0.3	1.0	0.6	0.0	-0.5	-0.7	-0.7
0.4	1.9	1.1	0.4	-0.1	-0.6	-0.8	0.4	0.8	0.5	-0.1	-0.5	-0.6	-0.7
0.5	1.3	0.7	0.0	-0.4	-0.7	-1.1	0.5	0.6	0.4	-0.2	-0.5	-0.6	-0.7
0.7	0.5	-0.1	-0.5	-0.6	-1.0	-1.3	0.7	0.6	0.3	-0.1	-0.5	-0.6	-0.6

⁎ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.

This flexibility of the price schedule at $T = 2$ makes the buyer better off compare to the nondiscriminatory schedule. For example, when $N = 6$ $\alpha = 0.4$, $s = 0.1$, SI outperforms EPC with the nondiscriminatory price schedule $R(t^1) = kt^1$(the difference between the buyer's expected total cost under EPC and SI is 0.6%) but EPC outperforms SI with the discriminatory schedule $R(t^1) = \kappa(t^1)t^1$ (the difference is -0.1%) (see table 4.3).

4.4.2 α under LI

Cost savings arising from learning by doing, economies of scale in production or other manifestations of synergies can occur in many forms. In section 4.1 we assumed that an incumbent's cost at $T = 2$ was drawn from the interval $[t^1 - \alpha, t^1]$, i.e., all suppliers faced the same absolute potential reduction in costs. An alternative cost framework is one where high cost suppliers have more potential for cost reduction (e.g., more 'low-hanging fruit'). We next consider the case where the maximum cost reduction is proportional to the supplier's type t^1, and as such, the cost range of an incumbent supplier is $[(1-\alpha)t^1, t^1]$, where α is the same for all suppliers.

(s increases from 0.1 to 0.3). For small s, $R(t^1) \to t^1 - \alpha$ for 'moderate' types when α is small, while $R(t^1) >> t^1 - \alpha$ when α is large. When α is small (e.g., $\alpha = 0.05$), the incumbent does not have a large cost advantage over entrants at $T = 2$; hence the buyer finds it optimal to set $R(t^1) = t^1 - \alpha$ for the moderate type suppliers in order to secure a price comparable to the cost of the lowest cost entrant at $T = 2$. When s is large ($s > 0.5$), the buyer wants to keep the incumbent for any α, and sets the optimal $R(t^1) = t^1 \ \ \forall t^1$, i.e., the set of 'moderate' suppliers is the null set.

Table 4.4: Comparison of non-discrim. EPC and SI with proportional α under LI1

$s = 0.1$							$\alpha = 0.1$						
$\alpha \diagdown N$	3	4	5	6	10	20	$s \diagdown N$	3	4	5	6	10	20
0.1	2.6	3.1	3.1	2.9	1.5	0.0	0.1	0.9	0.7	0.4	0.0	-0.6	-0.7
0.2	2.1	2.4	2.0	1.5	0.3	-0.5	0.2	0.6	0.3	-0.1	-0.4	-0.7	-0.8
0.3	1.5	1.3	1.0	0.7	-0.2	-0.6	0.3	0.4	0.0	-0.3	-0.6	-0.7	-0.8
0.4	0.9	0.7	0.4	0.0	-0.6	-0.7	0.4	0.3	-0.1	-0.4	-0.6	-0.7	-0.8
0.5	0.5	0.2	-0.3	-0.4	-0.7	-0.8	0.5	0.0	-0.2	-0.4	-0.6	-0.6	-0.8
0.7	-0.5	-0.8	-0.7	-0.8	-1.1	-1.1	0.7	-0.1	-0.1	-0.4	-0.6	-0.7	-0.8

⁕ The difference is measured by $\left(\frac{ETC_{EPC} - ETC_{SI}}{ETC_{SI}} \%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.
⁕⁕ the optimal $R^*(t^1) = t^1$ in above examples

Under the same market setting as in table 4.1, we found that (1) it is still optimal for the buyer to set $k = 1$ under a non-discriminatory EPC (see appendix C.2.1.) and (2)

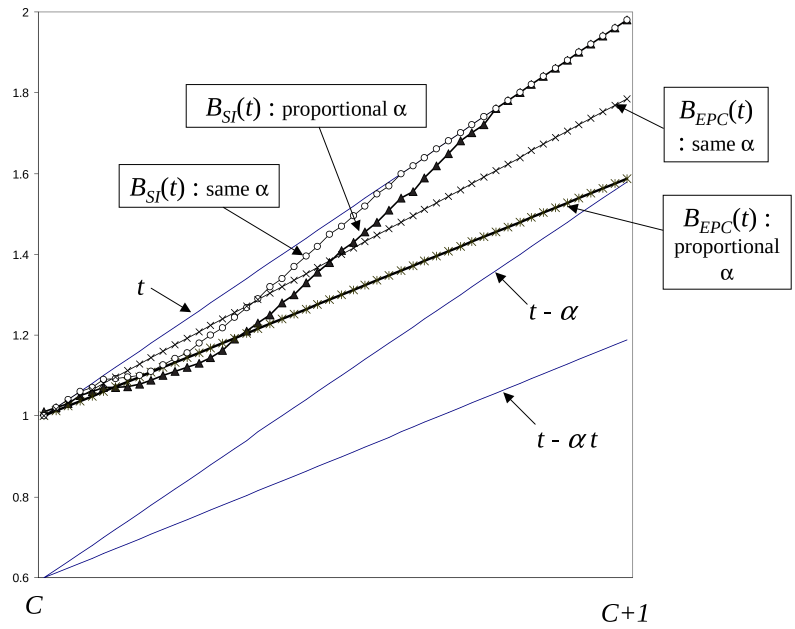


Figure 4.5: Bids under SI and EPC when the maximum possible cost reduction α is (1) same for all types and (2) proportional to the type ($N = 6$, $\alpha = 0.4$, $s = 0.1$)

observation 4.1 continues to hold (see table 4.4). Note that the performance of EPC is improved under this proportional cost reduction setting. Since high cost types face a larger potential decrease in cost at $T = 2$, this larger cost reduction coupled with the optimal schedule $R(t^1) = t^1$, implies that they bid more aggressively at $T = 1$ and this aggression cascades down to lower types, while under SI, the larger cost reduction for higher types can not be fully captured in bid at $T = 1$ due to the positive probability of not winning at $T = 2$. And hence, EPC's performance improves when the cost reduction is proportional to the bidder's type.

4.4.3 Cost distribution shift at $T = 2$

Another extension on our original model that we consider is a shift in the cost distribution at $T = 2$. In previous sections, we assumed that the distribution of costs for entrants at $T = 2$ stays same as in $T = 1$, i.e., $[C, C + 1]$, and only the incumbent redraws his cost from a new (lower) cost range. This is a valid assumption when industry costs are relatively stable over time, and/or there is only a single buyer in the market. However, the suppliers may be operating in a market where the cost of raw materials or technology increases or decreases over time. Alternatively, suppliers may be able to supply multiple buyers simultaneously and experience learning by doing through the production process for other buyers; however, the incumbent may still have an comparative cost advantage over the entrants due to buyer-specific learning-by-doing (e.g., leather seat example in the introduction). Both of these market effects can be modeled as a shift in the entrant's cost range at $T = 2$, i.e., $[C + \xi,\, C + 1 + \xi]$ ($\xi > (<) 0$).[9]

Under LI1 setting, the incumbent's cost at $T = 2$ would continue to be drawn from $[t^1 - \alpha, t^1]$, while the entrants' costs are drawn from $[C + \xi,\, C + 1 + \xi]$. Recall that the buyer uses effective bid rather than submitted bid for selecting the winning supplier at $T = 2$, due to the presence of the switching cost s; the shift in the entrants' cost distribution ξ plays a

[9]Under the NLI setting, the incumbent does not lock in his previous cost at $T = 2$ and, like entrants, must redraw his 'baseline cost' at $T = 2$. If the industry experiences a shift in costs, it effects both the incumbent and the entrants, and their cost distributions at $T = 2$ become $[C - \alpha + \xi,\, C + 1 - \alpha + \xi]$ and $[C + \xi,\, C + 1 + \xi]$, respectively. Since both the incumbent's and the entrants' costs shifts upward/downward by ξ, these shifts canceled out each other and hence our analysis and results in section 3.3 carry over.

Table 4.5: Comparisons of EPC and SI with entrant's cost shifts under LI1

			$\xi = -0.5$		$\xi = 0$		$\xi = 0.5$	
N	α	s	Δ_{ETC}	k	Δ_{ETC}	k	Δ_{ETC}	k
3	0.3	0.3	5.5	1.00	1.5	1.00	0.8	1.00
6	0.1	0.4	3.4	0.96	1.2	1.00	1.1	1.00
6	0.4	0.1	3.6	0.80	0.6	1.00	-0.4	1.00
10	0.2	0.3	3.1	0.90	-0.1	1.00	-0.2	1.00
10	0.3	0.2	3.0	0.85	-0.3	1.00	-0.5	1.00
20	0.1	0.5	1.0	1.00	-0.1	1.00	-0.1	1.00
20	0.5	0.1	1.7	0.76	-0.8	1.00	-0.8	1.00

$\divideontimes$ Δ_{ETC} is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.
$\divideontimes\divideontimes$ $R^*(t^1) = kt^1$
$\divideontimes\divideontimes\divideontimes$ $\xi = -0.5$: cost shifts downward, $\xi = 0.5$: cost shifts upward.

similar role and offers us some intuition as to the robustness of our results.

When the entrants' cost distribution shifts upward (i.e., $\xi > 0$), it is the same as an increase in s. The buyer would still find it optimal to set $k = 1$ and offer the contract $R(t^1) = t^1$, for now the incumbent is even more attractive to the buyer when compared to the case of $\xi = 0$. Similarly, when $\xi > 0$, EPC becomes more attractive than SI to the buyer . When the entrants' cost distribution shifts downward (i.e.,$\xi < 0$), $R(t^1) = t^1$ may no longer be optimal. For example, when $N = 6$, $\alpha = 0.4$, $s = 0.1$, the % cost difference of between EPC and SI is 0.6% when $\xi = 0$. It decreases to -0.4% when $\xi = 0.5$ (cost shift upward), and increases to 3.6% when $\xi = -0.5$ (cost shift downward) - see table 4.5 for more examples.

CHAPTER V

PROCUREMENT MECHANISMS UNDER LOCK-IN SETTING 2 (LI2)

5.1 Model

In the previous chapter 4 (LI1), the buyer makes her future payment based on the supplier's bid at $T = 1$. Under this setting, supplier t knows that the buyer sets the reserve price based on his bid (i.e., his reported cost type) at $T = 1$ if he wins at $T = 1$. Thus, the supplier takes into account his future payment as well as his winning chance at $T = 1$ when he reports his type at $T = 1$ with trying to capture some information rent. For example, if the supplier t reports his type as $\tau(> t^1)$ and wins the auction at $T = 1$, the reserve price is set to be τ under SI and the eroding price $R(\tau)$ under EPC.

However, in some market, buyers is able to audit their suppliers and make any future payments based on the supplier's actual observed costs, not his bid at $T = 1$. Under this setting, the supplier's true type t^1 (even if he reported his type as τ at $T = 1$) will be revealed to the buyer, and hence the reserve price is set to be t^1 under SI and the eroding price $R(t^1)$ under EPC.

Note that under both cases, the resulting reserve price r^2 in equilibrium is t^1 (i.e., the difference between LI1 and LI2 is the bidder's strategic behaviors at $T = 1$).[1]

In this chapter, we study LI2 in which the buyer audits the supplier's cost after the production (we compare LI1 and LI2 in chapter 5.5). All settings are the same as in LI1

[1] The difference becomes clearer if we compare supplier t's payoff when he wins at $T = 1$ and 2 and is paid by the reserve price r^2 (when $C + s \geq t < C + 1 - \alpha$). Under LI1 and LI2, payoffs are as follows.

$$\text{payoff under LI1} = \int_{t-\alpha}^{t} \int_{\max[\tau,\, C+s]}^{C+1+s} \int_{\tau}^{C+1} [B(w) + \tau - t - t_I^2]\, f_{(1:N-1)}(w) f_{(1:N-1)}(w_E^2) f_I(t_I^2)\, dw\, dw_E^2\, dt_I^2$$

$$\text{payoff under LI2} = \int_{t-\alpha}^{t} \int_{C+s}^{C+1+s} \int_{\tau}^{C+1} [B(w) + t - t - t_I^2]\, f_{(1:N-1)}(w) f_{(1:N-1)}(w_E^2) f_I(t_I^2)\, dw\, dw_E^2\, dt_I^2$$

See appendix D and F for the supplier's entire payoff function.

(see section 4.1), except the followings[2] :

- Under EPC, $R(t^1)$ is based on the supplier's actual cost at $T = 1$, and hence bidder t's bid at $T = 1$ can not affect $R(t^1)$ at $T = 2$.
- Under SI, reserve price r^2 is set to be t^1, the incumbent's cost at $T = 1$.

Similar to LI1, we solve for the equilibrium bidding strategies and the buyer's expected total costs under EPC and SI and proceed to compare these two mechanisms. Also notice that we are focusing on the uniform distribution[3].

5.2 Equilibrium Bidding Strategy and the Optimal Eroding Price $R(t^1)$ under EPC

We first start to solve for the equilibrium bidding strategy at $T = 1$ under EPC[4].

PROPOSITION 5.1. *In combination with the second period bids outlined in equation (2.1), the following constitutes a subgame perfect symmetric equilibrium bidding strategy under EPC,*

$$\begin{aligned} B^1_{EPC}(t^1) \quad &= \quad t^1 - \underbrace{\int_{t^1-\alpha}^{R(t^1)} F_I(x)\,dx}_{E[\Pi^2_{EPC}(t^2_I);R(t^1)]} \qquad\qquad (5.1) \\ &\quad + \underbrace{\left[1 - F_I(R(t^1))\right] \int_{C}^{C+1} \left[1 - F_{(1:N-2)}(x)\right] F(x)\,dx}_{E[\Pi^2_{EPC}(t^2_E);R(t)]} \qquad \forall\ t^1 \end{aligned}$$

Proof. see appendix E.1 □

Bidder t shades his bid by amount of the expected profit at $T = 2$ if he becomes an incumbent $E[\Pi^2_{EPC}(t^2_I); R(t^1)]$ (i.e., the incumbent supplier never experiences a negative profit at $T = 2$ because he has a right to reject $R(t^1)$ if $R(t^1)$ is less than his cost at $T = 2$).

[2]Note that LI2 is similar to NLI in the sense that the eroding price schedule and the reserve price are explicitly set by the buyer, not affected by the supplier. However, under NLI, this is because of the supplier's cost structure, not the fact that the payments are based on the observed actual cost.

[3]The derivation of equilibrium bids under EPC and SI applies to any general distribution.

[4]Same as in the previous model (LI1), an incumbent bids his true cost from his new cost distribution $[t^1 - \alpha, t^1]$, while other entrants submit their true types redrawn from $[C, C + 1]$ at $T = 2$.

However, this force is countered by the opportunity to participate in the second auction as an entrant with a new set of costs $E[\Pi^2_{EPC}(t^2_E); R(t^1)]$[5].

Given the bidding strategy, we can characterize the optimal non-discriminatory eroding price schedule $R(t^1) = kt^1$, and the corresponding bid and the buyer's expected total cost as follows.

PROPOSITION 5.2. *When $R(t^1) = t^1$ (i.e., $k = 1$), the buyer's expected total cost is minimized. Given this, the bidding strategy of a bidder t and the buyer's expected total cost are as follows.*

$$B^1_{EPC}(t^1) = t^1 - \left[t^1 - E[t^2_I]\right] \tag{5.2}$$

$$ETC_{EPC}(R^*(t)) = E[t^2_{I(2:N)}] + E[t^1_{(1:N)}] \tag{5.3}$$

Proof. see appendix E.2 and E.3. □

It is optimal for the buyer to set $R(t^1) = t^1$ which the incumbent always accepts at $T = 2$. Since the supplier t continues to supply the buyer if he is selected at $T = 1$, his only concern at $T = 1$ is to win, and hence he submits the most aggressive bid at $T = 1$, which is $E[t^2_I]$ (i.e., the payment at $T = 2$ and his cost at $T = 1$ are canceled out - see equation (5.2)). Note that the bid at $T = 1$ depends only on the amount of α, i,e., any supplier shades his bid by the same amount of the expected value of α below his cost.

The buyer's expected total cost $ETC_{EPC}(R(t^1))$ is given by $E[B^1_{EPC}(t^1)_{(2:N)}] + P_1 \times E[R(t^1)_{(1:N)}] + P_0 \times E[B^2_{EPC}(t^2)_{(2:N-1)} + s]$, where P_1 is the probability that the incumbent accepts $R(t^1)$, and $P_0 = 1 - P_1$. Under the optimal eroding price $R(t^1) = t^1$, the buyer's cost at $T = 1$ and 2 are the expectation of the second lowest bid at $T = 1$ ($E[t^2_{I(2:N)}]$) and the expectation of the lowest $R(t^1)$ ($E[t_{(1:N)}]$ - the buyer pays the incumbent's cost at $T = 1$), respectively.

[5]Note that this is same as $B^1_{EPC}(t^1)$ under LI1 except the last term with $R'(t^1)$ (equation (4.1) in chapter 4). This explains how the bidding behavior changes if the bidder's bid at $T = 1$ can not influence on the future payment.

5.3 Equilibrium Bidding Strategy and the buyer's expected total cost under SI

Under LI1 (in chapter 4), we were unable to derive a closed form solution for the bids at $T = 1$ under SI. The major reason of it is that the buyer sets the reserve price based on the supplier's bid at $T = 1$. When the buyer uses the supplier's actual cost at $T = 1$ instead of his bid, we can derive the closed bid function at $T = 1$ as follows.

PROPOSITION 5.3. *In combination with the second period bids outlined in equation (2.1), the following constitutes a subgame perfect equilibrium bidding strategy under SI,*

$$B^1_{SI}(t^1) = \begin{cases} (i)\ if\ C \le t^1 < C+s, \\ \quad t^1 - \underbrace{\left[t^1 - E[t^2_I]\right]}_{E[\Pi^2_{SI}(t^2_I)]} \\ (ii)\ if\ C+s \le t^1 < \min[C+s+\alpha, C+1], \\ \quad t^1 - \underbrace{\left[t^1 - E[t^2_I] - \int_{C+s}^{t^1} F_{E(1:N-1)}(x)\,(1-F_I(x))\,dx\right]}_{E[\Pi^2_{SI}(t^2_I)]} \\ \quad\quad + \underbrace{\left[\int_{C+s}^{t^1} \left(1 - F_{E(1:N-2)}(x)\right) F_E(x) F_I(x)\,dx\right]}_{E[\Pi^2_{SI}(t^2_E)]} \\ (iii)\ if\ \min[C+s+\alpha, C+1] \le t^1 < C+1, \\ \quad t^1 - \underbrace{\left[t^1 - E[t^2_I] - \int_{t^1-\alpha}^{t^1} F_{E(1:N-1)}(x)\,(1-F_I(x))\,dx\right]}_{E[\Pi^2_{SI}(t^2_I)]} \\ \quad\quad + \underbrace{\left[\int_{C+s}^{t-\alpha} \left(1 - F_{E(1:N-2)}(x)\right) F_E(x)\,dx + \int_{t^1-\alpha}^{t^1} \left(1 - F_{E(1:N-2)}(x)\right) F_E(x) F_I(x)\,dx\right]}_{E[\Pi^2_{SI}(t^2_E)]} \end{cases} \tag{5.4}$$

Proof. see appendix F.1. □

Due to the positive switching cost, the buyer considers the new supplier only if the reduced price is greater than the cost of switching suppliers. Thus, it is convenient to use the effective bid(cost); the entrant's effective bid is the positive shift of his submitted bid at $T = 2$ by amount of switching cost s (i.e., the distribution of the effective cost is F_E : $F_E(C + s) = 0$, $F_E(C + 1 + s) = 1$), and the incumbent's effective bid is the same as his submitted bid from the distribution F_I.

Different from the bid under EPC, a supplier shades(inflates) his bid below(above) his cost differently according to the relative position of costs between an incumbent and entrants: (i) supplier t's cost at $T = 2$, if he wins at $T = 1$, is less than the effective cost of an entrant ($t^1 < C + s \Rightarrow t_I^2 < C + s$), (ii) the supplier's cost as an incumbent and the entrant's effective cost is partly overlapped ($t^1 - \alpha \leq C + s < t^1$), and (iii) the supplier's cost as an incumbent lies in the entrant's effective cost range ($t^1 - \alpha > C + s$). When $t^1 < C+s$, the supplier t always wins and is paid by the reserve price t^1 at $T = 2$. Thus, his only incentive is to win at $T = 1$, and hence submits the most aggressive bid. However, as the incumbent's and the entrant's costs become overlapped, two effects forces the supplier to inflate his bid (i.e., his bid is less aggressive than the case (i)); (1) the supplier's profit as an incumbent decreases (i.e., the payment is combination of the reserve price and the lowest entrant's cost which is lower than the reserve price), and (2) the probability that an entrant wins at $T = 2$ and the expected profit as an entrant increase. These effects become larger when a supplier is higher type (i.e., the profit as an incumbent $E[\Pi_{SI}^2(t_I^2)]$ decreases and the profit as an entrant $E[\Pi_{SI}^2(t_E^2)]$ increases from (i) to (iii)).

Given the bidding strategy, the buyer's expected total cost ETC_{SI} is given by $E[B_{SI}^1(t^1)_{(2:N)}] + E[\min(\tilde{B}_{SI}^2(t^2)_{(2:N)}, r^2)]$, where $\tilde{B}_{SI}^2(t^2)_{(2:N)}$ is the effective bid, (i.e., if an entrant defines the payment, $\tilde{B}_{SI}^2(t^2)_{(2:N)} = B_{SI}^2(t^2)_{(2:N)} + s$). Given the bid in the proposition 5.3, it is as follows.

PROPOSITION 5.4. *Given the bidding strategy as in proposition 5.3, the buyer's expected* *total cost under SI is as follows.*

$$\begin{aligned} ETC_{SI} &= E[t^2_{I(2:N)}] + E[t^1_{(1:N)}] \\ &- \int_{C+s}^{C+1} \int_{\max[C+s,\, t-\alpha]}^{t} F_{E(1:N-1)}(x)\,[1 - F_I(x)]\,\big[f_{(1:N)}(t) - f_{(2:N)}(t)\big]\; dx\, dt \\ &+ \int_{C+s}^{C+1} \int_{\max[C+s,\, t-\alpha]}^{t} F_I(x)\,\big[F_{E(1:N-1)}(x)F(t) - F_{E(2:N-1)}(x)\big] \left(\frac{f_{(1:N)}(t)}{1-F(t)}\right) dx\, dt \\ &+ \int_{C+s+\alpha}^{C+1} \int_{C+s}^{t-\alpha} \big[F_{E(1:N-1)}(x)F(t) - F_{E(2:N-1)}(x)\big] \left(\frac{f_{(1:N)}(t)}{1-F(t)}\right) dx\, dt \end{aligned} \tag{5.5}$$

5.4 Analysis

In this section, we compare EPC and SI. For easy and clear insights, we use various numerical examples. We begin by first discussing the difference between bids under EPC and SI, and then proceed to compare the expected total costs of the buyer. After that, we discuss several variations from our original models : new sets of entrants, discount factor, and bundling. As we will see, our original models and results are easily applied to any of these variations.

5.4.1 Comparison of EPC and SI

Under SI, a bidder t submits less aggressive bid as his type(cost) becomes higher. This, combined with the fact that $B^1_{EPC}(t^1) = B^1_{SI}(t^1)$ when $t^1 \le C+s$(equation (5.2) and (5.4)), gives us the first comparison between EPC and SI.

LEMMA 5.1. *Suppliers bid more (or at least equally) aggressively under EPC than under SI, i.e.,* $B^1_{EPC}(t^1) \le B^1_{SI}(t^1)$, $\forall\, t^1$.

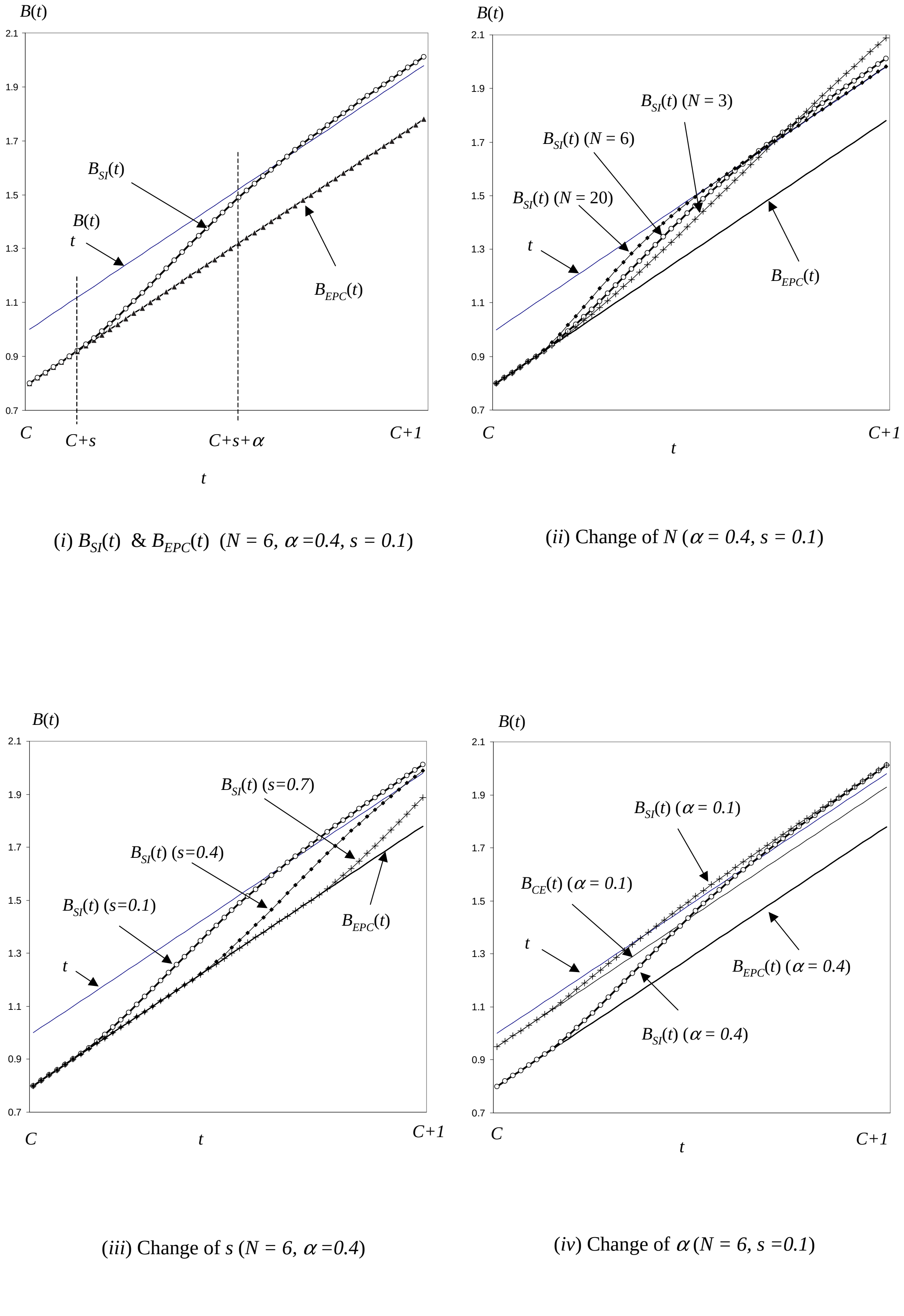
Figure 5.1 (i) shows $B^1_{EPC}(t^1)$ and $B^1_{SI}(t^1)$ when $N = 6$, $\alpha = 0.4$, and $s = 0.1$.

Recall that $B^1_{EPC}(t^1)$ does not change with N or s, while it does with α (i.e., bids shift downward by the same amount of $E[\alpha]$ when α increases). However, $B^1_{SI}(t^1)$ changes as any of market settings changes, and does so differentially (i.e., not by the same amount for all type). The low types ($t^1 \le C+s$) always submit the most aggressive bids under any market settings (N, α, or s). When N increases, moderate types ($C+s < t^1 \le C+s+\alpha$) tend to increase their bids (i.e., become less aggressive), while high types ($t^1 < C+s+\alpha$) actually decrease their bids (i.e., becomes more aggressive) - see figure 5.1 (ii). Note that both profits as an incumbent and an entrant decrease with N, i.e., entrants become more competitive and the difference between the first and the second lowest bids decreases. For a moderate type bidder, this reduced profit forces him to increase his bid as N increases. However, this reduced profit at $T = 2$, combined with the fact that the probability of winning at $T = 2$ is very small, forces the high type bidder to not inflate his bid above his cost at $T = 1$ (i.e., his bid stays nearer his cost at $T = 1$ as N increases; e.g. when $N = 20$, $B^1_{SI}(t^1) = t^1$ for high types - figure 5.1 (*ii*)). As s and α increase, an incumbent becomes competitive, and hence bidders in both moderate and high types become more aggressive to win at $T = 1$. When s increases, the range of low type and moderate type increase, and hence larger portion of bidder types submits the most aggressive bid (i.e., the range of $t^1 < C + s$ increases) - see figure 5.1 (*iii*). When α increases, both $B^1_{EPC}(t^1)$ and $B^1_{SI}(t^1)$ decrease - see figure 5.1 (*iv*).

Opposite to the case of the bid, the buyer's expected cost at $T = 2$ under SI is always less than or at least equal to that under EPC since the buyer is always able to select the most effective bidder at $T = 2$ under SI (this also implies the more aggressive bid at $T = 1$ under EPC than that under SI, i.e., the bidder's expected profit at $T = 2$ under EPC is greater than that under SI). However, the degree of the aggressiveness of bid at $T = 1$ under EPC tends to dominate the degree of the reduced payment at $T = 2$ under SI, and hence we have the following observation.

OBSERVATION 5.1. *In general, the buyer's expected total cost under EPC is less than that under SI. Conversely, SI outperforms EPC only when α and s are very small.*

Table 5.1 and 5.2 illustrate the difference of the buyer's expected total costs under EPC and SI. As in table 5.2, $ETC_{EPC} > ETC_{SI}$ when both α and s are very small (e.g. $\alpha = s = 0.01$) and N is not too small ($N > 5$). EPC outperforms SI best when N and s are small and α is large. Note that this is the case where the difference of the second lowest bid at $T = 1$ under SI and EPC is large. That is, when N (or s) is small, the second lowest bid would be among the moderate or high type (if N (or s) is large, it would be among the low type ($t^1 < C + s$) whose bid is same as in EPC). When α is large, the degree of aggression



(i) $B_{SI}(t)$ & $B_{EPC}(t)$ ($N = 6, \alpha = 0.4, s = 0.1$)

(ii) Change of N ($\alpha = 0.4, s = 0.1$)

(iii) Change of s ($N = 6, \alpha = 0.4$)

(iv) Change of α ($N = 6, s = 0.1$)

Figure 5.1: Bids under EPC and SI in various markets

Table 5.1: Comparison of non-discrim. EPC ($k = 1$) and SI under LI2 (1)

$s = 0.1$							$\alpha = 0.4$						
$\alpha \setminus N$	3	4	5	6	10	20	$s \setminus N$	3	4	5	6	10	20
0.1	-2.3	-1.3	-0.9	-0.7	-0.5	-0.2	0.1	-3.2	-2.5	-2.2	-1.9	-1.2	-0.4
0.2	-2.6	-1.8	-1.4	-1.2	-0.8	-0.3	0.2	-2.5	-1.9	-1.5	-1.2	-0.5	-0.1
0.3	-3.0	-2.2	-1.8	-1.6	-1.1	-0.4	0.3	-1.8	-1.3	-0.9	-0.7	-0.2	0.0
0.4	-3.2	-2.5	-2.2	-1.9	-1.2	-0.4	0.4	-1.1	-0.7	-0.5	-0.3	0.0	0.0
0.5	-3.5	-2.8	-2.5	-2.2	-1.3	-0.4	0.5	-0.6	-0.4	-0.2	-0.1	0.0	0.0
0.6	-3.7	-3.1	-2.7	-2.3	-1.4	-0.4	0.6	-0.3	-0.1	-0.1	0.0	0.0	0.0
0.7	-3.9	-3.3	-2.9	-2.5	-1.5	-0.5	0.7	-0.1	0.0	0.0	0.0	0.0	0.0
0.8	-4.1	-3.5	-3.0	-2.6	-1.6	-0.5	0.8	0.0	0.0	0.0	0.0	0.0	0.0
0.9	-4.3	-3.6	-3.2	-2.8	-1.6	-0.5	0.9	0.0	0.0	0.0	0.0	0.0	0.0

⋇ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\,\%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.

Table 5.2: Comparison of non-discrim. EPC ($k = 1$) and SI under LI2 (2)

$\alpha = s = 0.01$											
N	3	4	5	6	7	8	9	10	20	30	50
	-2.4	-0.8	-0.2	0.1	0.2	0.3	0.3	0.3	0.2	0.1	0.0

of bid decreases more rapidly (check the different slope of the moderate types with different α - see figure 5.1(iv)). This difference of bids at $T = 1$ makes the dominated effect on the buyer's expected total cost.

5.4.2 New Set of Entrants

In our original model, we assumed that the set of entrants at $T = 2$ are the same as in $T = 1$, i.e., the buyer faces exactly same entrants at $T = 2$. In the other side, the buyer may face totally different entrants at $T = 2$. These two, a same set and a new set of entrants, are the extreme cases. However, as we shall see, these two cases share many similar aspects. Hence, by observing the case of a new set of entrants and comparing it with the case of the same set of entrants, we fully expect that procurement settings situated between these two (i.e., the model in which the buyer faces partly new entrants and partly same entrants as in $T = 1$) will also yield the similar results.

So what would (or would not) change when the buyer faces the new set of entrants instead of the same set of them? The independent cost distribution of entrants when the buyer faces the same set of them indicates the following lemma.

LEMMA 5.2. *The buyer's expected costs at $T = 2$ are the same with new and same set of entrants under both procurement mechanisms, EPC and SI.*

However it is not the case in the buyer's expected cost at $T = 1$. With the new set of entrants, a bidder has no opportunity to participate in and possibly win the auction at $T = 2$, if he loses at $T = 1$. This makes a bidder always shade his bid below his cost at $T = 1$ (i.e., the profit as an entrant which forces him to inflate his bid does no longer exist). Thus, the bids at $T = 1$ under EPC and SI are the same as in equation (5.1) and (5.4) without $E[\Pi^2_{EPC}(t^2_E); R(t^1)]$ and $E[\Pi^2_{SI}(t^2_E)]$ terms respectively (i.e., bids with new set of entrants are more aggressive than those with same set of entrants because the terms which force bidders to inflate their bids disappear).

PROPOSITION 5.5. *When the buyer faces a new set of entrants at $T = 2$, she minimizes her expected total cost when $R^*(t^1) = t^1$ (i.e., $k = 1$), and hence her expected total costs under EPC are the same with new and same set of entrants.*

Proof. see appendix E.2.1 and E.3.1 □

By similar procedure as in the same set of entrants, it is optimal for the buyer to set $R^*(t^1) = t^1$ $(k = 1)$ under EPC when there are a new set of entrants at $T = 2$. Thus, the bidding strategy at $T = 1$ under EPC, $B^1_{EPC}(t^1)$ does not change under either a new or a same set of entrants, so does the buyer's expected cost at $T = 1$. Combined with the lemma 5.2, the buyer's expected total cost under EPC remains the same with the new or the same set of entrants.

Thus the only difference between the new and the same set of entrants is $B^1_{SI}(t^1)$. By

Table 5.3: Comparison of non-discrim. EPC ($k = 1$) and SI under LI2 (1) : new set of entrants

$s = 0.1$							$\alpha = 0.4$						
$\alpha \setminus N$	3	4	5	6	10	20	$s \setminus N$	3	4	5	6	10	20
0.1	-0.4	-0.3	-0.3	-0.4	-0.4	-0.2	0.1	-2.0	-2.0	-1.9	-1.8	-1.2	-0.4
0.2	-1.0	-1.0	-1.0	-1.0	-0.8	-0.3	0.2	-1.7	-1.6	-1.4	-1.2	-0.5	-0.1
0.3	-1.6	-1.5	-1.5	-1.4	-1.0	-0.4	0.3	-1.3	-1.1	-0.8	-0.6	-0.2	0.0
0.4	-2.0	-2.0	-1.9	-1.8	-1.2	-0.4	0.4	-0.9	-0.7	-0.4	-0.3	0.0	0.0
0.5	-2.5	-2.4	-2.2	-2.0	-1.3	-0.4	0.5	-0.5	-0.3	-0.2	-0.1	0.0	0.0
0.6	-2.8	-2.7	-2.5	-2.2	-1.4	-0.4	0.6	-0.3	-0.1	-0.1	0.0	0.0	0.0
0.7	-3.1	-2.9	-2.7	-2.4	-1.5	-0.5	0.7	-0.1	0.0	0.0	0.0	0.0	0.0
0.8	-3.4	-3.2	-2.8	-2.5	-1.6	-0.5	0.8	0.0	0.0	0.0	0.0	0.0	0.0
0.9	-3.6	-3.4	-3.0	-2.7	-1.6	-0.5	0.9	0.0	0.0	0.0	0.0	0.0	0.0

⁕ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\ \%\right)$; the negative terms imply $ETC_{EPC} < ETC_{SI}$.

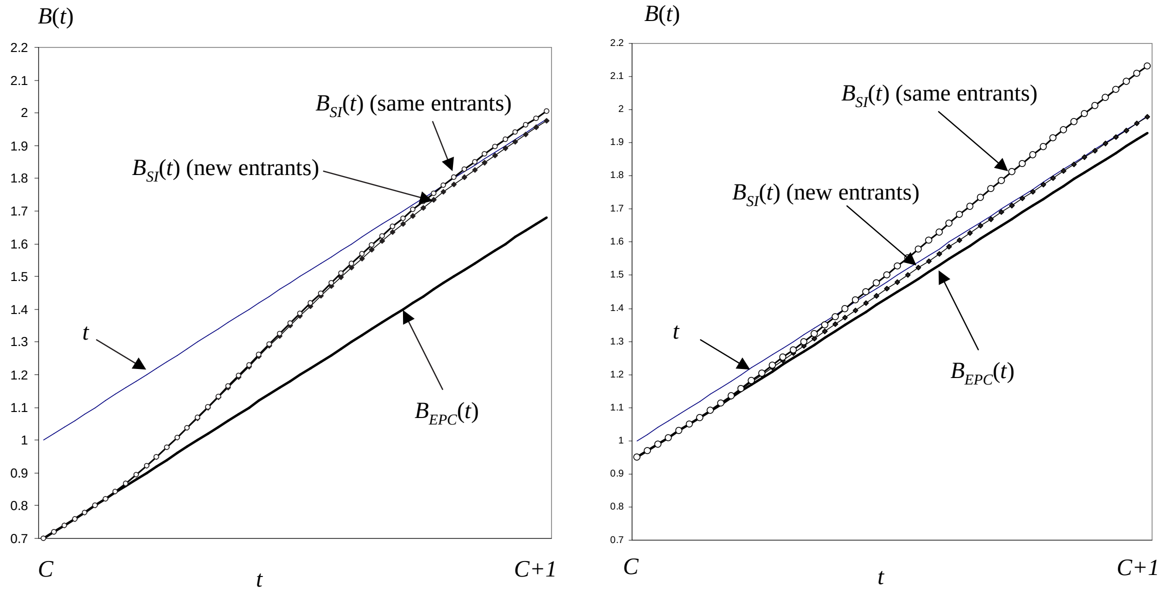
Table 5.4: Comparison of non-discrim. EPC ($k = 1$) and SI under LI2 (2) : new set of entrants

$\alpha = s = 0.01$											
N	3	4	5	6	7	8	9	10	20	30	50
	0.5	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.3	0.1	0.0

the fact that the bids with the new set of entrants are more aggressive than those with the same set the following proposition is straightforward.

PROPOSITION 5.6. *The performance of SI with the new set of entrants improves compared to that with the same set of entrants.*

It is clear if we compare table 5.1 (with the same set of entrants) and 5.3 (with the new set of entrants). Even if the performance of SI improves, EPC still outperforms SI as with the same set of entrants in most market settings. However, the difference between EPC and SI decreases. Note that the improvement of SI increases as N, α and s decrease. For



(i) N = 6, α = 0.6, s = 0.1

(ii) N = 3, α = 0.1, s = 0.1

Figure 5.2: Change of bids with same and new set of entrants

example, under the market settings of $N = 6$, $\alpha = 0.6$ and $s = 0.1$, and $N = 3$, $\alpha = s = 0.1$, EPC outperforms SI by 2.3% with the same set of entrants, while EPC outperforms SI by 2.2% in formal case($N = 6$, $\alpha = 0.6$ and $s = 0.1$) and by 0.4% in latter case($N = 3$, $\alpha = s = 0.1$) with the new set of entrants. Again, this tendency comes from the different bidding behaviors at $T = 1$ under SI. That is, under SI, the bid at $T = 1$ become very aggressive if there is no chance of the second auction compared to the bid with the chance, when N, α and s are small. Recall that when α or s is small, the inflation of bid is greater under SI with same entrants (see figure 5.1 (iii) and (iv)). This inflation disappears if the bidder has no additional chance for the auction at $T = 2$. This is also the case of small N, where higher types inflate their bid above their costs with same entrants. With large N, the expected profit as an incumbent as well as an entrant would be similar with the same entrants and new entrants, and hence the bidding strategy would not change a lot. See figure 5.2 for the difference of bidding strategies under SI when $N = 6$, $\alpha = 0.6$ and $s = 0.1$, and $N = 3$, $\alpha = s = 0.1$ to see why the latter market setting improves the performance of SI better than the formal setting.

5.4.3 Discount Factor δ

Another interesting variance from our original model is a discount factor. In our original model, we assumed that the future cost ($T = 2$) is same important as the present cost ($T = 1$) to the buyer. In this section, we show how the results would change if a discount factor δ is less than 1 (in the original model, δ might be considered as 1). The buyer's expected total cost under procurement mechanism M (EPC or SI) with the discount factor δ is then,

$$ETC_M = EC_M^1 + \delta EC_M^2, \tag{5.6}$$

where EC_M^T is the buyer's cost under mechanism M at time T. As δ decreases, the degree of the expected cost at $T = 2$ becomes less significant. Thus, the impact of the first period becomes relatively larger. Then how would the buyer's cost at $T = 1$ (i.e. the suppliers' bids at $T = 1$) change?

Given a discount factor δ, supplier t's bid at $T = 1$ under procurement mechanism M is as follows.

$$B_M^1(t^1) = t^1 - \delta E[\Pi_M^2(t_I^2)] + \delta E[\Pi_M^2(t_E^2)], \quad 0 < \delta < 1 \tag{5.7}$$

Under EPC, the value of δ does not affect the choice of the optimal R^* since δ is multiplied to all $R(t^1)$ in ETC_{EPC}.

LEMMA 5.3. *With considering the discount factor $\delta < 1$, the buyer minimizes her expected total cost when $R^*(t^1) = t^1$ (i.e. $k = 1$).*

As a result, supplier t shades his bid at $T = 1$ below his cost by amount of $\delta E[\Pi_M^2(t_I^2)]$ which is less than the case of $\delta = 1$. Recall that all suppliers shade their bids below

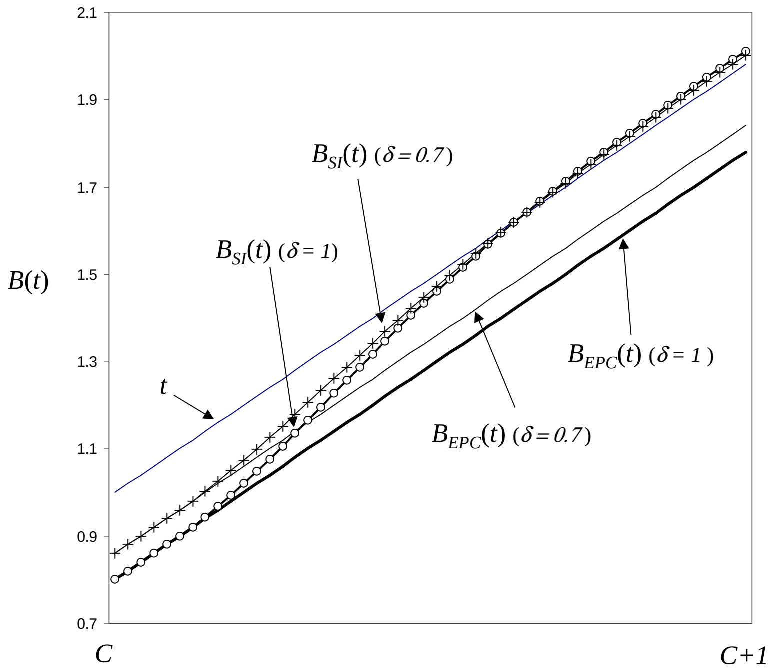


Figure 5.3: Change of bids when δ =1 and 0.7 ($N = 6$, $\alpha = 0.4$, $s = 0.1$)

their costs by the same amount under EPC. Under SI, however, δ makes the impact of the expected profit at $T = 2$ on supplier t's bid at $T = 1$ differentially according to supplier's type (see figure 5.3 for the change of bids when $\delta = 1$ and 0.7). For a low type supplier, the impact of δ is more significant on his expected profit as an incumbent $E[\Pi_M^2(t_I^2)]$ than his profit as an entrant $E[\Pi_M^2(t_E^2)]$. Thus, his bid would be less aggressive than that with $\delta = 1$ case. Especially, for a supplier $t^1 < C+s$, he shades his bid by amount of $\delta E[\Pi_M^2(t_I^2)]$ which is same as under EPC. Recall that the high cost at $T = 1$ forces a high supplier type to inflate the bid above his cost when $\delta = 1$. With $\delta < 1$, the second chance is less significant to him, and hence he inflates the bid less than that when $\delta = 1$ (see figure 5.3). That is, as δ decreases, all supplier types inflate their bids from their bids with $\delta = 1$ under EPC, while some supplier types (i.e. high types) may shade more from their bids with $\delta = 1$ under SI.

OBSERVATION 5.2. *As the buyer values more the current contract than the future contract (i.e. δ decreases), the performance of SI improves.*

As a result of different bidding behaviors, the degree of increase of the expected cost

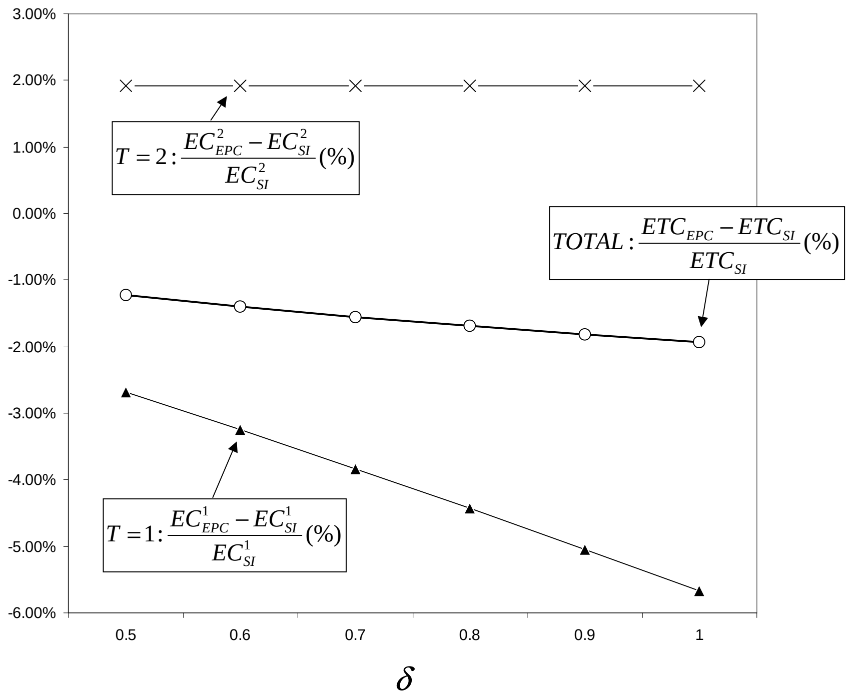


Figure 5.4: Change of the buyer's expected total costs under EPC and SI with δ ($N = 6$, $\alpha = 0.4$, $s = 0.1$)

of the buyer at $T = 1$ under SI is less than that under EPC, and hence SI becomes more attractive compared to the case of $\delta = 1$. Figure 5.4 illustrates the comparison of the buyer's expected costs under EPC and SI as δ changes. The difference between EPC and SI decreases as δ decreases. Note that the difference of the buyer's expected cost at $T = 2$ is the same regardless of δ since the expected cost at $T = 2$ changes at the same rate either EPC and SI, while the difference at $T = 1$ decreases when δ decreases.

5.4.4 Bundling

Under EPC and SI, the buyer commits to a short term contract, in which she renews her contract after the first period with her incumbent or possibly a new supplier. Under EPC, it is optimal that the buyer keeps her incumbent at $T = 2$ ($R^*(t^1) = t^1$). That is, EPC with the optimal $R(t^1) = t^1$ becomes equivalent to the long term contract with the supplier who wins at $T = 1$. Then, does it mean that the buyer's optimal choice of $R(t^1)$ would still be t^1 if she commits a long term contract?

When the buyer commits a long term contract with eroding price schedule $R(t^1)$ (i.e. she bundles her eroding price contract), she auctions off both units at $T = 1$, and the lowest bidder wins the entire contract at different payments at $T = 1$ and 2 (i.e. the supplier t is paid by the lowest rejected bid for the first unit, and the eroding price $R(t^1)$ for the second unit). Note that in the long term contract, the winning supplier also commits to supply the buyer in both time periods even if his cost is greater than the eroding price at $T = 2$ ($t_I^2 > R(t^1)$).

The supplier t's bid at $T = 1$ would be $t^1 - \big(R(t^1) - E[t_I^2]\big)$; his only incentive is to win at $T = 1$, and hence he shades his bid below his cost by amount of the expected profit at $T = 2$. The cost of the buyer at $T = 2$ is $R(t^1)$ (no switching cost s is involved). Thus, the buyer's expected total cost is given by $E[t^1_{(2:N)}] - \Big(kE[t^1_{(2:N)}] - E[t^2_{I(2:N)}]\Big) + kE[t^1_{(1:N)}]$. The first three terms are the expected second lowest bid at $T = 1$ ($R(t^1) = kt^1$), and the last term is the expected cost of the buyer at $T = 2$ (i.e. the expected eroding price of an incumbent). From this, it is straightforward that $k = 1$ minimizes the buyer's expected total cost same as in the short term contract.

PROPOSITION 5.7. *When the buyer bundles the eroding price contract, she minimizes her cost with $R(t^1) = t^1$ (i.e. $k = 1$), and hence the bidding strategy of a bidder t and the buyer's expected total cost are the same as under the short term contract (i.e., EPC).*

The buyer may simply bundle the entire contract by using auction only, in which the buyer selects her supplier for both periods, and the payment of both units are determined by the lowest rejected price at $T = 1$. Since the winner of the auction supplies the buyer for whole two units at the same bid price, the nice truth-telling properties of a one-shot single-unit Vickrey auction carry over to this bundle format (i.e. the supplier t's bid at $T = 1$ is same as $\frac{t^1+t_I^2}{2}$). The buyer's expected total cost is as follows.

PROPOSITION 5.8. *When the buyer conducts the bundle auction(BA), the buyer's expected total cost is given by $E[t^1_{(2:N)}] + E[t^2_{I(2:N)}]$.*

Intuitively, BA can not outperform SI because the buyer fails to find more effective

Table 5.5: Comparison of Sequential and Bundle auctions under LI2 (1)

$s = 0.1$							$\alpha = 0.4$						
$\alpha \diagdown N$	3	4	5	6	10	20	$s \diagdown N$	3	4	5	6	10	20
0.1	6.8	6.5	5.9	5.3	3.6	2.1	0.1	6.3	5.6	4.9	4.4	3.1	2.1
0.2	6.6	6.1	5.4	4.8	3.3	2.0	0.2	7.1	6.3	5.6	5.1	3.8	2.4
0.3	6.4	5.8	5.1	4.6	3.2	2.0	0.3	7.9	7.0	6.3	5.7	4.2	2.5
0.4	6.3	5.6	4.9	4.4	3.1	2.1	0.4	8.6	7.6	6.8	6.1	4.4	2.5
0.5	6.2	5.4	4.8	4.3	3.1	2.1	0.5	9.1	8.0	7.1	6.3	4.4	2.5
0.6	6.1	5.4	4.7	4.2	3.1	2.1	0.6	9.5	8.2	7.2	6.4	4.4	2.5
0.7	6.1	5.3	4.7	4.2	3.2	2.2	0.7	9.7	8.3	7.2	6.4	4.4	2.5
0.8	6.1	5.3	4.7	4.2	3.2	2.3	0.8	9.8	8.3	7.3	6.4	4.4	2.5
0.9	6.2	5.4	4.7	4.3	3.3	2.3	0.9	9.8	8.4	7.3	6.4	4.4	2.5

⋇ The difference is measured by $\left(\frac{ETC_{SI}-ETC_{BA}}{ETC_{SI}}\%\right)$; BA is a bundle auction

Table 5.6: Comparison of Sequential and Bundle auctions under LI2 (2)

$\alpha = s = 0.01$											
N	3	4	5	6	7	8	9	10	20	30	50
	6.5	6.8	6.5	6.0	5.5	5.1	4.7	4.4	2.4	1.7	1.0

supplier at $T = 2$ ($\Rightarrow ETC_{BA} > ETC_{SI(orEPC)}$)[6]. Table 5.5 and 5.6 show some numerical comparisons between SI and BA. Note that the difference increases as N and α decrease, and s increases[7].

5.5 *Comparisons of LI1 and LI2 : Should the buyer audit the supplier's cost?*

The only difference between LI2 and LI1(chapter 4) is whether the supplier's bid at $T = 1$ directly affects the buyer's future price or not. Under LI1, the buyer sets her eroding price

[6]It is straightforward to see that $ETC_{EPC} < ETC_{BA}$ because $E[t_{(1:N)}] < E[t_{(2:N)}]$.

[7]In appendix H, we discuss the bundling contracts under NLI and LI1. Different from the case under LI2, the buyer's expected total costs under the bundling eroding price contract and the bundling auction are the same in either model, NLI and LI1. Especially, under LI1, the choice of the optimal $R(t^1)$ (i.e. k) does not matter to the buyer ; any k yields the same output to the buyer.

under EPC and reserve price under SI based on the supplier's bid at $T = 1$, while she sets those based on the supplier's actual cost at $T = 1$ under LI2. As we have seen, this made the significantly different impact on the comparisons between EPC and SI : the bidder's bidding behavior and the performance of the procurement mechanisms.

Under the same market setting (N, α and s), the performance under SI tends to be better than that under EPC under LI1 (i.e. the buyer's expected total cost under SI is lower than that under EPC), while the opposite happens under LI2 (see table 4.2 and 5.1). Note that the buyer's expected total costs at $T = 2$ are the same under LI1 and LI2 in equilibrium. Thus it is clear that the different results owe to the different bidding behaviors at $T = 1$ under LI1 and LI2 (see figure 4.3 for bids under LI1 and 5.1 under LI2).

Recall that under LI2, $B_{EPC}(t^1) < B_{SI}(t^1)$ for $t^1 > C + s$, and $B_{EPC}(t^1) = B_{SI}(t^1)$ for the low type supplier ($t^1 \leq C + s$), i.e. the buyer's expected cost at $T = 1$ under EPC is always lower than that under SI (see lemma 5.1). Under LI1, the high chance of winning at $T = 1$, combined with the fact that the payment at $T = 2$ would be decided based on the bid at $T = 1$ (i.e. the future price is based on the bid (i.e. the reported cost), not the actual cost), forces the low type supplier to bid less aggressively than the high type supplier under both EPC and SI. Note that the bid under SI can be less than that under EPC for moderate or even low type suppliers (see figure 4.3 (i) and (ii)). Hence, it is not always true that the bid under EPC is less than or same as that under SI.

The buyer's expected cost at $T = 1$ is determined by the second lowest bid and this is more likely set from one of the moderate (or high) group suppliers ($t^1 > C + s$). Thus the difference between EPC and SI becomes larger under LI2, while it is not always true under LI1. Therefore, under LI2, it is more likely that EPC outperforms SI in many market setting, while the opposite can be true under LI1. That is, under LI2, the lower expected cost of the buyer at $T = 1$ under EPC dominates the lower cost at $T = 2$ under SI. Under LI1, the difference of the buyer's cost at $T = 1$ under EPC and SI is not large enough to dominate the lower buyer's cost at $T = 2$ under SI, or even there is the case where the buyer's cost at $T = 1$ under SI can be lower than that under EPC.

This suggests us that, if it is not too costly for the buyer to audit the supplier for his cost after the production (and before the second contract), the long term relationship with a supplier (EPC) would be a better choice rather than the short term contracts (SI).

For example, when $N = 6$, $\alpha = 0.4$ and $s = 0.1$, the relationships of the buyer's expected total costs between EPC and SI are as follows (ETC_M^{LIx} denotes the buyer's expected total cost under the mechanism M in the model LIx where $x \in \{1, 2\}$).

$$\begin{aligned} ETC_{SI}^{LI1} &< ETC_{EPC}^{LI1} \\ ETC_{EPC}^{LI2} &< ETC_{SI}^{LI2} \end{aligned}$$

Since the buyer can extract more information rent from the supplier by observing his true cost, she can lower her cost under LI2 compared to that under LI1. Thus, the buyer's expected total costs under different mechanisms can be ranked as follows.

$$ETC_{EPC}^{LI2} < ETC_{SI}^{LI2} < ETC_{SI}^{LI1} < ETC_{EPC}^{LI1}$$

That is, if the cost of auditing the supplier is less than $ETC_{SI}^{LI1} - ETC_{EPC}^{LI2}$, then it would be worthwhile for the buyer to commit the long term contract with the supplier.

CHAPTER VI

SEQUENTIAL AUCTIONS WITH OPTIMAL RESERVE PRICES (BASED ON NLI)

So far we examined the procurement mechanisms which are frequently used in practice. Different from EPC, where the buyer explicitly sets one of the payments at $T = 2$ by eroding price (the other is the price set by the auction among entrants, if any), the submitted bids at $T = 2$ determines the buyer's payment at $T = 2$ under SI. However, the buyer uses a *reserve* price under SI for trying to exert some control over the price at $T = 2$. Under our SI model (in NLI), it was $r^2 = C + 1 - \alpha$. While this choice of the reserve price is very straightforward and intuitive (i.e., the buyer will not pay above the maximum incumbent's cost at $T = 2$), it is obvious that the buyer can save more money by choosing the reserve price more systematically. First, the buyer may set the reserve price at $T = 1$ by trade off between the low payment and the possibility to not have any supplier (so possibly less than $C + 1$). Recall that the buyer has no control over the price at $T = 1$ under SI. Under SI, the reserve price at $T = 1$ can be considered as $C + 1$, at which the buyer select the reserve to maximize the number of suppliers who were eligible to participate at $T = 2$. Second, it is well-known that the discriminatory reserve price rule should be applied for asymmetric bidders (under SI, the reserve price at $T = 2$ are the same for both an incumbent and entrants).

In this chapter, we study the selection of the *optimal* reserve prices in each of period under the sequential procurement setting and make comparisons with other mechanisms, SI and EPC.

6.1 Optimal reserve prices model(OPT)

The model is same as SI under NLI : the buyer auctions off short term contract and selects the winning supplier in each of two periods via a Vickrey auction. Different from SI, the buyer sets the reserve prices in both periods (possibly different reserve prices for an incumbent, entrants, etc). We assume that the buyer announces the reserve prices at $T = 1$ and 2 in the beginning of $T = 1$.[1] If there is no supplier who satisfies the reserve price in either periods, the buyer should incur some opportunity cost, i.e, the buyer might purchase the product at the noncompetitive price. In our model, we assume that if she fails to select her winner from N suppliers, she incurs her opportunity cost as amount of $C + 1$ at $T = 1$, and $C + s + 1$ at $T = 2$.

To obtain the optimal solution of minimizing her expected total cost, the buyer has four decision variables $r = (r^1, r^2_0, r^2_I, r^2_E)$, where $r^1 \in [C, C+1]$, $r^2_0, r^2_E \in [C+s, C+1+s]$, and $r^2_I \in [C-\alpha, C+1-\alpha]$. r^T is a reserve price at time T. Depending on the reserve price at $T = 1$ (r^1), the buyer faces two possible situations at $T = 2:$ (1) there is no incumbent supplier at $T = 2$, i.e., the buyer does not purchase from the suppliers at $T = 1$, and (2) there is an incumbent at $T = 2$. In the first case, the buyer needs to determine a reserve price at $T = 2$ (r^2_0) against N symmetric bidders at $T = 2$, while she should determine two (discriminatory) reserve prices in the second setting, one for the incumbent (r^2_I) and another for the $N - 1$ entrants (r^2_E).

Since bidders are rational, they incorporate the expected profit at $T = 2$ when they submit their bids at $T = 1$. Furthermore, the reserve prices at $T = 2$ as well as at $T = 1$ affect the equilibrium bidding strategy at $T = 1$, as defined in the following equation.

$$B^1_{OPT}(t^1) = t^1 - E[\Pi^2(t^2_I)] + (1 - p_0(t^1))E[\Pi^2_1(t^2_E)] + p_0(t^1)E[\Pi^2_0(t^2_E)], \tag{6.1}$$

where p_0 is the probability that no supplier satisfies the buyer's reserve price at $T = 1$. Bidder t shades/inflates his bid by amount of the expected profit at $T = 2$. The bidder t's profit as an incumbent at $T = 2$ is $E[\Pi^2(t^2_I)]$. Bidder t's expected profit as an entrant can be either $E[\Pi^2_1(t^2_E)]$, if there is an incumbent supplier at $T = 2$ (and hence $N - 1$ entrants), or $E[\Pi^2_0(t^2_E)]$ if there is no incumbent (i.e., N symmetric entrants).

Note that p_0 is a function of t^1 as well as r^1, i.e., for a bidder whose cost at $T = 1$ is less

[1]Since there is no additional information after the first auction, the outcome would be the same whether the buyer announces the reserve prices at $T = 2$ before or after the second auction.

than the buyer's reserve price r^1 ($t^1 \leq r^1$), the probability that there exists an incumbent at $T = 2$ is always 1 ($p_0 = 0$). The buyer pays $\min[B^1_{(2:N)}(t^1), r^1]$ if $t^1 \leq r^1$, and the opportunity cost otherwise. Thus, the bidding strategy we are interested in is only for the type no greater than r^1, and hence $B^1_{OPT}(t^1(\leq r^1)) = t^1 - E[\Pi^2(t^2_I)] + E[\Pi^2_1(t^2_E)]$ (see appendix J.1 for detailed formulations of $E[\Pi^2(t^2_I)]$ and $E[\Pi^2(t^2_E)]$).

Given the bidding strategy at $T = 1$ and 2 (the bidding strategy at $T = 2$ is true cost-telling by Vickery rule), the buyer finds her optimal reserve prices which minimize her expected total cost, $ETC_{OPT}(r)$.

$$ETC_{OPT}(r) = EC^1(r^1, r^2_I, r^2_E) + (1 - p_0(r^1))EC^2_1(r^2_I, r^2_E) + p_0(r^1)EC^2_0(r^2_0), \tag{6.2}$$

where EC^1 is the expected cost at $T = 1$, EC^2_I is the expected cost at $T = 2$ if there is an incumbent ($I = 1$), or no incumbent ($I = 0$). Note that EC^1 is not a function of r^2_0 (see appendix J.2 for detailed formulations of EC^1, EC^2_I).

Since r^2_0 affects only $EC^2_0(r^2_0)$, combined with the fact that N entrants are symmetric, the optimal r^2_0 is straightforward by Myerson[34].

LEMMA 6.1. *The optimal reserve price at $T = 2$, when there exists no incumbent, is obtained from the following equation.*

$$r^2_0 + \frac{F_E(r^2_0)}{f_E(r^2_0)} = C + 1 + s \tag{6.3}$$

The allocation and the payment rules in EC^1 and EC^2_0 are simple in the sense that the bidders are symmetric, i.e., the lowest bidder t wins, if his bid is no greater than the reserve price, with the payment of min[reserve price, the lowest rejected bid], or the buyer incurs the opportunity cost, if there is no winner. If there exists an incumbent at $T = 2$, bidders become asymmetric. To obtain EC^2_1, for consistency, the allocation and the payment rules are set as follows;

- **allocation rule** A winning supplier is the lowest bidding supplier among bidders whose costs are no greater than the corresponding reserve prices at $T = 2$.

- **payment rule** The payment is min[reserve price of the winning supplier, the lowest rejected bid, if any] (see Appendix J.2 for detail).

Different from r_0^2, the other three variables r^1, r_I^2, and r_E^2 are interdependent (equation (6.2), for explicit expression of $ETC_{OPT}(r)$, see appendix J.2), and hence they do not give an explicit closed-form solution. Hence, in the next section, we solve it numerically by approximation method.

6.2 Analysis

We assume that costs are drawn from uniform distributions. We approximate the uniform distribution of a supplier's cost, the reserve prices r^1, r_I^2, and r_E^2 by discretizing the corresponding intervals of $[C, C+1]$, $[C-\alpha, C+1-\alpha]$, and $[C+s, C+1+s]$ by increments of $\frac{1}{n}$ (results illustrated in this section are based on experiments with $n = 200$). To capture a wide spectrum of market settings, we varied the experimental settings, i.e., the values of α, s, and N, as in table 6.1.

Table 6.1: Parameters used in numerical examples

C	$1 : t^1 \sim U[1, 2]$
N	2, 3, 4, 5, ..., 10, 20, 30, 40, 50
α	0.01, 0.03, 0.05, 0.1, 0.2, ..., 0.5
s	0.01, 0.03, 0.05, 0.1, 0.2, ..., 0.5

We first observe the characters of the optimal reserve prices in each period under OPT and then proceed to the comparisons with SI and EPC.

It is well know from Myerson[34] and Riley and Samuelson[35] that the optimal reserve price in the model of symmetric bidders in one period does not depend on the number of bidders, N (in our model, r_0^2 is the case ; see r_0^2 in table 6.2). However, if the model deviates from it, i.e., if there are (1) multiple periods (observation 6.1), and/or (2) asymmetric bidders (observation 6.2 and 6.3), we found that the choice of the optimal reserve price becomes a function of N.

Table 6.2: Optimal reserve prices

Market			Reserve Prices				Probability of a bidder's cost being below reserve price, $Pr(t_i^T \leq r_i^T)$			
N	α	s	r^1	r_0^2	r_I^2	r_E^2	$t^1 \leq r^1$	$t_E^2 \leq r_0^2$	$t_I^2 \leq r_I^2$	$t_E^2 \leq r_E^2$
2	0.2	0.1	1.690	1.600	1.800	1.105	0.690	0.5	1	0.005
4			1.620	1.600	1.800	1.290	0.620	0.5	1	0.190
6			1.585	1.600	1.800	1.305	0.585	0.5	1	0.205
8			1.565	1.600	1.800	1.315	0.565	0.5	1	0.215
10			1.555	1.600	1.800	1.315	0.555	0.5	1	0.215
20			1.540	1.600	1.800	1.325	0.540	0.5	1	0.225
4	0.05	0.1	1.570	1.600	1.950	1.330	0.570	0.5	1	0.230
	0.1		1.585	1.600	1.900	1.320	0.585	0.5	1	0.220
	0.2		1.620	1.600	1.800	1.290	0.620	0.5	1	0.190
	0.3		1.660	1.600	1.700	1.265	0.660	0.5	1	0.165
	0.4		1.705	1.600	1.600	1.235	0.705	0.5	1	0.135
	0.5		1.750	1.600	1.500	1.210	0.750	0.5	1	0.110
4	0.2	0.05	1.605	1.550	1.800	1.255	0.605	0.5	1	0.205
		0.1	1.620	1.600	1.800	1.290	0.620	0.5	1	0.190
		0.2	1.660	1.700	1.800	1.365	0.660	0.5	1	0.165
		0.3	1.705	1.800	1.800	1.435	0.705	0.5	1	0.135
		0.4	1.750	1.900	1.800	1.510	0.750	0.5	1	0.110
		0.5	1.795	2.000	1.800	1.580	0.795	0.5	1	0.080

OBSERVATION 6.1. *Under OPT, when N decreases, and/or α and s increase, r^1 increases.*

Note that in two periods, the optimal choice of r^1 depends on the cost shifts at $T = 2$ by α and s (i.e., the incumbent's and the entrant's costs) as well as N. Table 6.2 illustrates the change of r^1 for different values of N, α and s. For example, when N decreases from 20 to 2, r^1 increases from 1.54 to 1.69, and when α increases from 0.05 to 0.5, it increases from 1.57 to 1.75[2]. When N is small, and α and s are large, the winner at $T = 1$ has a large

[2]We show the examples that the buyer considers more than half of the supplier types in table 6.2. However, with large N (e.g. 50), the buyer would consider less than half of types (47%).

(expected) cost advantage over his competitors at $T = 2$. And hence, the expected profit at $T = 2$ is large, which incents bidders to bid aggressively at $T = 1$. To maximize this competitive pressure, the buyer sets a high reserve, which in turn increases the probability that the buyer will incur a low cost at $T = 2$ by keeping an incumbent at $T = 2$ (as r^1 increases, p_0 decreases).

When the buyer faces asymmetric bidders (i.e., an incumbent and entrants), the optimal choices of reserve prices for difference bidders (r_I^2 and r_E^2) are as follows.

OBSERVATION 6.2. *Under OPT, for any market setting (N, α, s), the buyer discriminates against entrants and in favor of the incumbent, i.e., the buyer guarantees not to exclude any incumbent types from the auction at $T = 2$ and sets $r_I^2 = C + 1 - \alpha$, while she sets $r_E^2 < C + 1 + s$.*

By setting $r_I^2 = C + 1 - \alpha$, the buyer does not exclude any type of incumbent (i.e., $\Pr(t_I^2 > r_I^2) = 0$), while she finds it optimal to set a reserve that excludes some entrant types. For example, in table 6.2(last column), the buyer considers less than 30 % of entrant types.

The sequential nature of the bidding events reinforces the difference between r_I^2 and r_E^2. Suppose that there is only one auction event, and that the bidders are asymmetric : one bidder with a cost drawn from distribution F_I, and $N - 1$ bidders with their costs drawn from distribution F_E. The reserve prices for the incumbent and entrants are given by $t_i^2 + \frac{F_i(t_i^2)}{f_i(t_i^2)} = C + 1 + s$, $i = \{I, E\}$. Under the uniform distribution, $Pr(t_I^2 \le r_I^2) = \frac{1+\alpha+s}{2}$ and $Pr(t_E^2 \le r_E^2) = \frac{1}{2}$. Clearly, if $\alpha + s < 1$, $Pr(t_I^2 \le r_I^2) < 1$. The buyer finds it optimal to exclude only the high incumbent types, while she always excludes half of the entrant types. When this asymmetric auction becomes the second bidding event, as in our model, the buyer favors the incumbent even more by setting r_I^2 to its highest possible level, $C + 1 - \alpha$ ($Pr(t_I^2 \le r_I^2) = 1$), but become more strict towards entrants (by setting r_E^2 such that $Pr(t_E^2 \le r_E^2) < \frac{1}{2}$).

By allowing only a small set of entrant types to potentially displace the incumbent as her

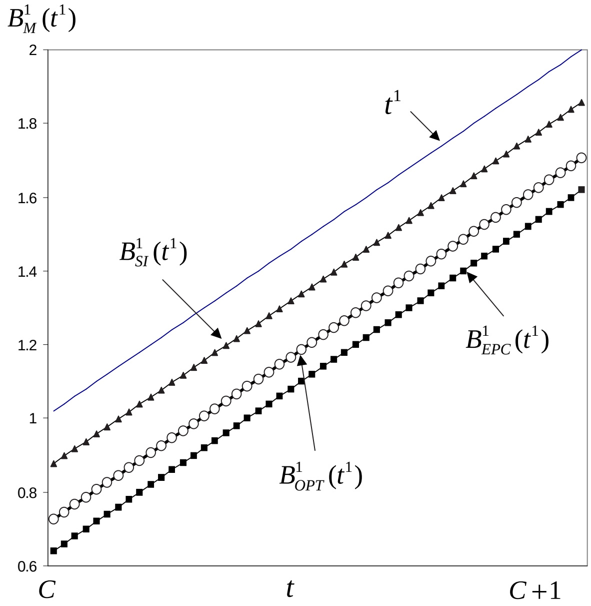
Table 6.3: Comparisons of the expected costs among EPC, SI, and OPT : $N = 4$, $\alpha = 0.2$, $s = 0.1$

Mechanisms (M)	EC_M^1	EC_M^2	ETC_M (total)	reserve	price	type portion§
EPC	1.020	1.674	2.693	r^1 $r_I^{2\dagger}$ $r_E^{2\ddagger}$	$2(=C+1)$ $1.683(=R^*)$ $2(=C+1)$	1 0.88 1
SI	1.258	1.425	2.682	r^1 r_I^2 r_E^2	$2(=C+1)$ $1.8(=C+1-\alpha)$ $1.8(=C+1-\alpha)$	1 1 0.7
OPT	1.104	1.536 $(EC_0^2 = 1.488)$ $(EC_1^2 = 1.537)$	2.640	r^1 r_0^2 r_I^2 r_E^2	1.62 1.6 $1.8(=C+1-\alpha)$ 1.29	0.62 0.5 1 0.19

§ portion of cost types which the buyer is willing to consider as a supplier - normalized as 1.
†, ‡ under EPC, the reserve price is conditional.

supplier at $T = 2$, the buyer might risk excluding a low-cost entrant at $T = 2$[3]. However, this risk is outweighed by the aggressive bidding behavior it induces at $T = 1$. To see this, we compare the expected cost at $T = 2$ if an incumbent exists under OPT with the expected cost at $T = 2$ under SI. The reserve prices at $T = 2$ under SI can be considered to be $r_I^2 = r_E^2 = C + 1 - \alpha$, and hence the only difference from OPT is r_E^2. For example, when $N = 4$, $\alpha = 0.2$, and $s = 0.1$, the expected costs at $T = 2$ with the existence of an incumbent(EC_1^2) are 1.537 under OPT and 1.425 under SI, respectively (see table 6.3). The optimal r_E^2 actually does not reduce the expected cost at $T = 2$ under OPT. However, combined with the choice of $r_I^2 = C+1-\alpha$ and r^1(observation 6.1), the choice of r_E^2 reduces the expected cost at $T = 1$ under OPT; the bidders bid very aggressively to win at $T = 1$ (see figure 6.1) because the expected profit at $T = 2$ as an incumbent is considerably larger than that as an entrant (compare the expected profit as an incumbent $E[\Pi^2(t_I^2)]$ and as an

[3]For example, consider the case where $r_E^2 < t_E^2 < t_I^2 < r_I^2$ (t_E^2 is an entrant' cost and t_I^2 is an incumbent cost at $T = 2$). The buyer would pay $\min[t_I^2, r_E^2]$ instead of r_I^2, if the buyer sets r_E^2 is slightly higher than t_E^2.



M	$E[\Pi_M^2(t_I^2)]$	$E[\Pi_M^2(t_E^2)]$	Δ^*
EPC	0.390	0.010	0.380
SI	0.168	0.026	0.142
OPT	0.302	0.008	0.294

$\Delta^* = E[\Pi^2(t_I^2)] - E[\Pi^2(t_E^2)]$,
$B_M^1(t^1) = t^1 - \Delta^*$

Figure 6.1: Bids under SI and OPT at $T = 2 :$ $N = 4$, $\alpha = 0.2$, $s = 0.1$

entrant $E[\Pi^2(t_E^2)]$ in figure 6.1). A reserve of $r_I^2 = C + 1 - \alpha$ also protects the buyer from incurring the opportunity cost at $T = 2$ $(= C + 1 + s)$ (there always exists one supplier whose cost is no greater than the reserve price at $T = 2$). In addition, by setting $r^1 < C+1$, the buyer cuts off high cost types at $T = 1$ (under SI, $r^1 = C + 1$, hence there is no cut-off for any types). In table 6.3, the expected costs at $T = 1$ are 1.104 under OPT (EC_{OPT}^1) and 1.258 under SI (EC_{SI}^1), respectively.

Under EPC, the reserve prices facing entrants at $T = 2$ are conditional in the sense that r_E^2 is only considered after the incumbent rejects the offered reserve price $r_I^2(= R^*)$. Different from OPT (or SI), the buyer may discriminate against an incumbent in the sense that the buyer considers a partial set of the incumbent type when $R^* < \hat{R}(< C + 1 - \alpha)$, while she does not exclude any type of entrants by (implicitly) setting $r_E^2 = C+1$ (recall that interpreting a EPC mechanism as a choice of reserves does not capture all of its dynamics. Under EPC, the incumbent faces a reserve of r_I^2, but is given first priority to accept/reject the contract. Only after he has rejected, the entrants have a chance to replace him. Hence there is no direct competition between the incumbent and entrants at $T = 2$.). From the illustrated example in table 6.3($N = 4$, $\alpha = 0.2$, $s = 0.1$), the optimal R is less than $C + 1 - \alpha$(the buyer considers the lowest 88% of incumbent types), while she accepts any type of an entrant. However, by the incumbent's first right to accept/refuse the contract at $T = 2$, the expected profit as an incumbent under EPC is greater than that under OPT. ($E[\Pi^2_{EPC}(t^2_I)] = 0.390 > 0.302 = E[\Pi^2_{SI}(t^2_I)]$ in figure 6.1)[4]. Thus, the bid under EPC would be more aggressive than that under OPT. However, these reserve prices under EPC, combined with the fact that the reserve prices are conditional, give the buyer a higher expected cost at $T = 2$ under EPC (1.674) compare with that under OPT (1.536).

While the buyer always sets $r^2_I = C + 1 - \alpha$ under any given market settings, the buyer is willing to consider a larger set of the entrant types when the entrant become more competitive as in the following observation (see table 6.2).

OBSERVATION 6.3. *When N increases, and/or α and s decreases, the optimal reserve for entrants at $T = 2$ increases.*

So far, we observed the characters of the optimal reserve prices under sequential auctions(OPT) and the dynamics of the reserve prices SI(EPC) and OPT, i.e., the change of a supplier's expected cost at $T = 2$ as an incumbent and an entrant by different reserve prices and the effect on the buyer's expected costs in each period. Given this, we can establish the expected cost difference between SI(EPC) and OPT as change of market settings (see table 6.4 and 6.5).

OBSERVATION 6.4. *When a value of any market setting (N, α or s) increase, the cost difference between SI (or EPC) and OPT decreases.*

As α and s increase, the entrants become less competitive, and hence the buyer would set the reserve price for entrants close to the lowest possible cost type ($r^2_E \simeq C + s$) (observation 6.3), while she sets r^1 close to $C + 1$ (observation 6.1). Thus, OPT becomes similar to SI (or EPC) ; with large α and s, there is a very small chance that an entrant wins the auction

[4]Suppose that supplier t's cost at $T = 2$ is 1.5. Under EPC, he would accepts $R^* = 1.683$ with the profit 0.183, while he might not win at $T = 2$ if there is an entrant whose cost is less than $r^2_E = 1.29$.

Table 6.4: Comparison of SI and OPT

$s = 0.1$						$\alpha = 0.2$					
$\alpha \diagdown N$	4	6	8	10	20	$s \diagdown N$	4	6	8	10	20
0.05	2.03	0.80	0.33	0.14	0.00	0.05	1.78	0.82	0.39	0.19	0.01
0.1	1.90	0.81	0.36	0.16	0.00	0.1	1.58	0.79	0.40	0.20	0.01
0.2	1.58	0.79	0.40	0.20	0.01	0.2	1.19	0.70	0.40	0.23	0.02
0.3	1.23	0.72	0.42	0.24	0.02	0.3	0.81	0.56	0.37	0.24	0.03
0.4	0.88	0.61	0.40	0.26	0.03	0.4	0.48	0.39	0.30	0.22	0.04
0.5	0.55	0.45	0.34	0.25	0.05	0.5	0.24	0.23	0.20	0.16	0.05

⁎ The difference is measured by $\left(\frac{ETC_{SI}-ETC_{OPT}}{ETC_{OPT}}\ \%\right)$

Table 6.5: Comparison of EPC and OPT

$s = 0.1$						$\alpha = 0.2$					
$\alpha \diagdown N$	4	6	8	10	20	$s \diagdown N$	4	6	8	10	20
0.05	2.51	1.33	0.72	0.43	0.10	0.05	2.24	1.37	0.79	0.49	0.10
0.1	2.37	1.34	0.75	0.45	0.10	0.1	2.00	1.33	0.80	0.50	0.10
0.2	2.00	1.33	0.80	0.50	0.10	0.2	1.42	1.21	0.80	0.52	0.11
0.3	1.48	1.26	0.83	0.54	0.11	0.3	0.83	1.01	0.75	0.52	0.12
0.4	0.90	1.10	0.81	0.57	0.13	0.4	0.43	0.69	0.63	0.49	0.13
0.5	0.48	0.79	0.72	0.56	0.15	0.5	0.18	0.31	0.41	0.39	0.14

⁎ The difference is measured by $\left(\frac{ETC_{EPC}-ETC_{OPT}}{ETC_{OPT}}\ \%\right)$

under SI (or EPC), and the buyer excludes all (or most of) the entrant under OPT. When $\alpha + s \geq 1$, SI (or EPC) converges to OPT.

When N increases, there exists less difference among an incumbent and entrants. From the observation 6.1 and observation 6.3, the buyer sets r^1 close to C (i.e., p_0 increases) and r_E^2 close to $C + s + 1$. That is, the buyer becomes less strict towards entrants who become competitive, and hence OPT becomes similar to SI (or EPC).

CHAPTER VII

CONCLUSION

Our main interests were : (i) to study the optimal design of an eroding price contract when suppliers experience learning by doing and (ii) to establish when/if a buyer is better off committing to a single supplier in return for the supplier offering her a more competitive price. We asked these questions in different cost frameworks, NLI , LI1, and LI2. We found that,

- Under both LI1 and LI2 where F follows a uniform distribution, it is optimal for the buyer to always guarantee her incumbent supplier a non-negative profit at $T = 2$ when using a non-discriminatory price schedule, i.e., $R(t^1) = t^1 \quad \forall t^1$. The buyer can further reduce her expected costs by using a discriminatory price mechanism. However, we found the additional cost savings to be very small, suggesting that the added complexity of designing of discriminatory EPC is not warranted.
- In contrast, the buyer may not find it optimal to guarantee the incumbent supplier her business under NLI (derived for general cost distribution F).
- Even in the presence of learning by doing, a buyer is often better off running sequential auctions with a reserve price, rather than limiting competition and contracting with a single supplier in the hopes of extracting a better future price. Our numerical examples show that the cost difference between the two mechanisms is very small under NLI, but can be substantial under LI1 and LI2.

Moreover, by extending models in various ways, we found that,

- Under LI1 (and LI2), the incumbent locks-in his cost at $T = 2$, and hence a buyer would be better off by not locking in a supplier for two periods and should instead run SI when the entrant's cost distribution shifts downward.

- Under NLI and LI1, the buyer is better off not bundling her contract,[1] while under LI2, the buyer is indifferent of using either the short-term or long-term contract when she uses an eroding price contract.

- The buyer's ability to audit the supplier's actual cost makes a significant impact on the performance of the procurement mechanisms. The buyer is better off by keeping the relationship with her incumbent if auditing is not too costly.

- The buyer can reduce her expected cost under the sequential auctions by optimally selecting the reserve prices. Especially, the buyer favors her incumbent against entrants by always accepting her incumbent (i.e., incumbent's reserve price is always set to be his highest cost).

It is important to note that we have labeled the cost reduction that occurs at $T = 2$ as a learning by doing effect. However, our analysis and results apply equally for similar manifestations of economies of scale. We also considered a setting where the buyer has a known demand of Q in each period. It is possible that the buyer has only an estimate of her demand in each period, $E[Q]$. However, as long as both the buyer and seller's are risk-neutral, our analysis and results carry over.

There are a number of ways to expand our models and test the robustness of our results. Especially, we suggest some interesting extensions of our model in the hopes of bridging our work with the existing works.

First, it would be interesting to extend our model to include settings where the suppliers can reduce their costs over time by exerting costly effort, i.e., the cost reduction no longer occurs 'naturally' but rather is the result of a supplier's conscious effort to reduce production costs (see Laffont and Tirole[23] for excellent survey and study of the investment as an effort of the cost reduction). Under such a framework, the buyer may use an EPC not only to try to 'capture' some of the cost savings that a supplier accrues, but also as a *catalyst* for the supplier to identify and undertake cost-saving effort/action and as a result, we conjecture that its performance will improve, relative to SI.

[1] See appendix H for bundled procurement mechanisms under NLI and LI1.

Second, we studied the case where the buyer uses a sole sourcing strategy, i.e., the buyer purchases an individual product from a supplier. Alternatively, some buyers prefer to have multiple sourcing strategies (e.g. when a buyer purchases several products and wants to keep two suppliers, each of them supplies some portion of the products - 50-50 or 80-20). Under such a setting, it would be interesting to extend our EPC model to solve the optimal eroding price schedules according to each supplier's type and further, the optimal portion of the business combined with the eroding price contract (Klotz and Chatterjee[19] studied the design of two sequential auctions when the buyer wants dual sourcing).

Third, we studied two-period model with N potential suppliers. By extending the periods ($T > 2$), we can proceed to find the optimal eroding price $R(t^{T-1}, T)$ under EPC, which has now an additional dimension of period T. Under the two-period model (i.e., LI1 and LI2), the optimal $R(t^1) = t^1$. However, it would no longer optimal for the buyer to always guarantee nonnegative profit to her incumbent if there are more than 2 periods. In addition, the buyer may want to renew the eroding price at every period after observing the bid or the incumbent's reaction for the given eroding price. This would be a good complement of Lewis and Yildirim[25] who studied the sequential auctions among 2 suppliers in infinite time horizon ($T = \infty$).

Finally (but not the last one), we would consider the capacity constraints in our model (Elmaghraby[13] studied the sequential auctions where two groups of suppliers exist; one group has a capacity constraint, while the other group has not). Depending on the supplier's capacity, the magnitude of the synergy (e.g. cost reduction due to learning by doing) as well we the bidding behavior would differ, and hence the buyer would set different eroding price $R_c(t^1)$ for a group with capacity c.

APPENDIX A

BIDDING STRATEGIES AND THE EXPECTED TOTAL COSTS UNDER EPC (IN CHAPTER 3 : NLI)

In this appendix, we obtain the bidding strategies and the expected total costs of the buyer under EPC. Without loss of generality, we analyze the bidding strategy of a bidder of type t at $T = 1$. Assume that all bidders, with the possible exception of bidder t, are using the same bid strategy $B(\cdot)$ at $T = 1$ and bid as their true types. Given this, we consider the profit maximization problem for bidder t. We define $\Pi(t, \tau)$ to be the expected profit of type t submitting bid as type τ at $T = 1$ ($\tau = t^1 \pm \delta$). In order for the bid function $B(\cdot)$ to be a symmetric equilibrium bid, the payoff of the bidder t should satisfy the first order condition at $\tau = t^1$, when we consider both $\tau < t^1$ and $\tau > t^1$. We present below our analysis when $\tau < t^1$; the results are identical when $\tau > t^1$.

A.1 The equilibrium bidding strategy of a bidder t

A supplier t has three possible positive payoffs streams : when he (a) wins at $T = 1$ and (finds it profitable to) accepts R at $T = 2$, (b) wins at $T = 1$ and rejects R at $T = 2$, and (c) loses at $T = 1$ and wins at $T = 2$. We define w^1 as the lowest bidder among $N - 1$ bidders at $T = 1$ (excluding t^1).

The supplier t's payoff is,

$$\begin{aligned}
\Pi(t,\tau) \quad = \quad & \int_{C-\alpha}^{R} \int_{\tau}^{C+1} [B(w^1) + R - t^1 - t_I^2] \, f_{(1:N-1)}(w^1) f_I(t_I^2) \, dw^1 \, dt_I^2 \\
& + \int_{R}^{C+1-\alpha} \int_{\tau}^{C+1} [B(w^1) - t^1] \, f_{(1:N-1)}(w^1) f_I(t_I^2) \, dw^1 \, dt_I^2 \\
& + \int_{R}^{C+1-\alpha} \int_{C}^{\tau} f_{(1:N-1)}(w^1) f_I(w_I^2) \, dw^1 \, dw_I^2 \\
& \times \int_{C}^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2) \, dw_E^2 \, dt_E^2
\end{aligned}$$

where t_S^T denotes the bidder t at time T as the status of S (i.e., I if he is an incumbent,

E if he is an entrant), w^1 is the lowest bidder among $N-1$ bidders excluding bidder t at $T=1$, w_S^2 denotes the bidder $w(\neq t)$ at $T=2$ as the status of S (i.e., I if he is an incumbent, E if he is an entrant).

The equilibrium bidding strategy by solving the first order condition (by replacing τ with t^1) is,

$$\begin{aligned} B_{EPC}^1(t^1) = t^1 \quad & - \quad \underbrace{\int_{C-\alpha}^{R} (R - t_I^2) f_I(t_I^2)\, dt_I^2}_{E[\Pi_{EPC}^2(t_I^2);R]} \\ & + \quad \underbrace{(1 - F_I(R)) \int_C^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2}_{E[\Pi_{EPC}^2(t_E^2);R]}, \\ \forall\ t^1 \end{aligned} \tag{A.1}$$

A.2 The optimal R^* and the buyer's expected total cost

Given the bidding strategy, the buyer's expected total cost is

$$\begin{aligned} ETC_{EPC}(R) &= E[B_{EPC}^1(t^1)_{(2:N)}] + P_1 \times R + P_0 \times E[B_{EPC}^2(t_E^2)_{(2:N-1)} + s] \\ &= E[t_{(2:N)}^1] - \int_{C-\alpha}^{R} (R - t_I^2) f_I(t_I^2)\, dt_I^2 \\ &\quad + (1 - F_I(R)) \int_C^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2 \\ &\quad + F_I(R) R + (1 - F_I(R)) \left(\int_C^{C+1} t_E^2 f_{(2:N-1)}(t_E^2) dt_E^2 + s \right) \end{aligned}$$

We find the optimal R by taking the first order condition, and prove that it is indeed a global solution.

$$\begin{aligned} \frac{\partial ETC_{EPC}(R)}{\partial R} &= -\underbrace{\int_{C-\alpha}^{R} f_I(t_I^2)\, dt_I^2}_{=F_I(R)} \\ &\quad - f_I(R) \int_C^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2 \\ &\quad + f_I(R) R + F_I(R) - f_I(R) \left(\int_C^{C+1} t_E^2 f_{(2:N-1)}(t_E^2) dt_E^2 + s \right) \\ &= f_I(R)(R - \Lambda) = 0, \end{aligned} \tag{A.2}$$

where $\Lambda = \int_C^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2 + \int_C^{C+1} t_E^2 f_{(2:N-1)}(t_E^2) dt_E^2 + s$. Let $\hat{R}$ be the solution of the equation (A.2).

Now, we check whether $\hat{R}$ is a global optimal value. To do this, we can simply check the sign of the first derivative of $ETC_{EPC}(R)$.

$$\frac{\partial ETC_{EPC}(R)}{\partial R} = \begin{cases} < 0, & \text{if } R < \Lambda \\ = 0, & \text{if } R = \Lambda \\ > 0, & \text{if } R > \Lambda \end{cases}$$

In the range of R ($[C-\alpha, C+1-\alpha]$), the optimal R^* is determined as follows.

$$R^* = \begin{cases} \hat{R}, & \text{if } C-\alpha \le \hat{R} < C+1-\alpha \\ C+1-\alpha, & \text{if } \hat{R} \ge C+1-\alpha \end{cases}$$

(Note that $R^* = C-\alpha$ can not happen, since $R^* > C-\alpha(\because E[B^2_{EPC}(t^2_E)_{(2:N-1)} + s] > C+s > C-\alpha)$)

Given the optimal R^*, the buyer's expected total cost is,

(1) If $R^* = C+1-\alpha$,

$$ETC_{EPC}(R) = E[t^1_{(2:N)}] + E[t^2_I]$$

(2) If $R^* = \hat{R}$,

$$\begin{aligned} ETC_{EPC}(R) &= E[t^1_{(2:N)}] - \int_{C-\alpha}^{\hat{R}} (\hat{R} - t^2_I) f_I(t^2_I)\, dt^2_I + F_I(\hat{R})\hat{R} + (1 - F_I(\hat{R}))\hat{R} \\ &= E[t^1_{(2:N)}] + E[t^2_I] - \int_{\hat{R}}^{C+1-\alpha} t^2_I f_I(t^2_I)\, dt^2_I + (1 - F_I(\hat{R}))\hat{R} \\ &= E[t^1_{(2:N)}] + E[t^2_I] - \int_{\hat{R}}^{C+1-\alpha} 1 - F_I(x) dx \end{aligned}$$

A.3 proof of $E[\Pi^2_{EPC}(t^2_I); R^*] > E[\Pi^2_{EPC}(t^2_E); R^*]$ under the uniform distribution

We show that given the optimal R^* under the uniform distribution, the supplier t always shades his bid at $T = 1$ below his cost t^1 even if he considers the expected profit as an entrant at $T = 2$ by which he would inflate his bid.

First, when $R^* = C+1-\alpha$, supplier t always shades his bid since he always accepts R^*. When $R^* = \hat{R}$, the supplier t's expected profits at $T = 2$ as an incumbent and an entrant are as follows.

$$
\begin{aligned}
E[\Pi^2_{EPC}(t^2_I);\hat{R}] &= \int_{C-\alpha}^{\hat{R}} (\hat{R}-t^2_I) f_I(t^2_I)\, dt^2_I = \int_{C-\alpha}^{\hat{R}} F_I(t^2_I)\, dt^2_I \\
E[\Pi^2_{EPC}(t^2_E);\hat{R}] &= (1-F_I(\hat{R})) \int_{C}^{C+1} \int_{t^2_E}^{C+1} [w^2_E - t^2_E] f_{(1:N-2)}(w^2_E) f(t^2_E)\, dw^2_E\, dt^2_E
\end{aligned}
$$

Under the uniform distribution, the difference is as follows.

$$
\begin{aligned}
& E[\Pi^2_{EPC}(t^2_I);\hat{R}] - E[\Pi^2_{EPC}(t^2_E);\hat{R}] \\
= \; & \left[\frac{(\hat{R}-C+\alpha)^2}{2}\right] - \left[\frac{1+C-\alpha-\hat{R}}{N(N-1)}\right] \\
= \; & \frac{3}{2}\left(\frac{1}{N(N-1)}\right)^2 + \frac{2}{N(N-1)}\left(\frac{2}{N}+s+\alpha\right) + \frac{1}{2}\left(\frac{2}{N}+s+\alpha\right)^2 \\
& -\frac{1}{N(N-1)}, \quad \left(\leftarrow \hat{R} = \frac{1}{N(N-1)} + \frac{2}{N} + C + s\right) \\
> \; & \frac{3}{2}\left(\frac{1}{N(N-1)}\right)^2 + \frac{2}{N(N-1)}\left(\frac{2}{N}\right) + \frac{1}{2}\left(\frac{2}{N}\right)^2 - \frac{1}{N(N-1)} \\
& (\because \alpha + s > 0) \\
= \; & \left(\frac{1}{N(N-1)}\right)^2 \left[N^2 + N - \frac{1}{2}\right] \\
> \; & 0 \quad (\because N \geq 3)
\end{aligned}
$$

APPENDIX B

BIDDING STRATEGIES AND THE EXPECTED TOTAL COSTS UNDER SI (IN CHAPTER 3 : NLI)

B.1 The equilibrium bidding strategy of a bidder t

Supplier t has five possible payoffs : when he (a) wins in both periods and an entrant defines his payment at $T = 2$, (b) wins in both periods and the buyer pays her reserve price $r^2 = C + 1 - \alpha$, (c) wins only at $T = 1$, (d) wins only at $T = 2$ and an entrant defines his payment and (e) wins only at $T = 2$ and the incumbent defines his payment. Note that the buyer switches her suppliers only when at least one of the entrants' cost is lower than incumbent's cost minus the switching cost s. Without loss of generality, we can obtain the payoff of supplier t by shifting the cost range of an entrant by s (we use the same method to obtain the expected total cost at $T = 2$ of the buyer).

(<u>case1</u>) $\alpha + s \geq 1$

When supplier t wins at $T = 1$, he also wins at $T = 2$ with probability of 1 and the payment is the buyer's reserve price, $C + 1 - \alpha$ (all entrants' bids $\geq C + s \geq C + 1 - \alpha$). If he loses at $T = 1$, the incumbent w_I will continue to win at $T = 2$ with probability of 1, and there is no second chance of winning for the bidder t. Therefore bidder t's payoff is given by

$$\Pi(t, \tau) = \int_{C-\alpha}^{C+1-\alpha} \int_{\tau}^{C+1} [B(w^1) + r^2 - t^1 - t_I^2]\, f_{(1:N-1)}(w) f_I(t_I^2)\, dw\, dt_I^2$$

The equilibrium bidding strategy is,

$$B_{SI}^1(t^1) = t^1 - [r^2 - E[t_I^2]],\ r^2 = C + 1 - \alpha$$

(<u>case2</u>) $\alpha + s < 1$

The payoff function of supplier t, when he bids as type τ is,

$$\begin{aligned}
\Pi(t,\tau) = \int_{C-\alpha}^{C+1-\alpha} \int_{\max[C+s,\, t_I^2]}^{C+1-\alpha} \int_{\tau}^{C+1} \quad & [B(w^1) + w_E^2 - t^1 - t_I^2] f_{(1:N-1)}(w^1) \times \\
& f_{E(1:N-1)}(w_E^2) f_I(t_I^2)\, dw^1\, dw_E^2\, dt_I^2 \\
+ \int_{C-\alpha}^{C+1-\alpha} \int_{r^2}^{C+1+s} \int_{\tau}^{C+1} \quad & [B(w^1) + r^2 - t^1 - t_I^2] f_{(1:N-1)}(w^1) \times \\
& f_{E(1:N-1)}(w_E^2) f_I(t_I^2)\, dw^1\, dw_E^2\, dt_I^2 \\
+ \int_{C+s}^{C+1-\alpha} \int_{C+s}^{t_I^2} \int_{\tau}^{C+1} \quad & [B(w^1) - t^1] f_{(1:N-1)}(w^1) \times \\
& f_{E(1:N-1)}(w_E^2) f_I(t_I^2)\, dw^1\, dw_E^2\, dt_I^2 \\
+ \int_{C}^{\tau} \int_{C+s}^{C+1-\alpha} \int_{C+s}^{w_I^2} \int_{t_E^2}^{w_I^2} \quad & [w_E^2 - t_E^2]\, f_{(1:N-1)}(w^1) \times \\
& f_{E(1:N-2)}(w_E^2) f_E(t_E^2) f_I(w_I^2)\, dw_E^2\, dt_E^2\, dw_I^2\, dw^1 \\
+ \int_{C}^{\tau} \int_{C+s}^{C+1-\alpha} \int_{C+s}^{w_I^2} \int_{w_I^2}^{C+1+s} \quad & [w_I^2 - t_E^2]\, f_{(1:N-1)}(w^1) \times \\
& f_{E(1:N-2)}(w_E^2) f_E(t_E^2) f_I(w_I^2)\, dw_E^2\, dt_E^2\, dw_I^2\, dw^1
\end{aligned}$$

The equilibrium bidding strategy of bidder t is

$$\begin{aligned}
B_{SI}^1(t^1) \;=\; & t^1 \\
& - \underbrace{\Bigg[\int_{C-\alpha}^{C+1-\alpha} \int_{\max[C+s,\, t_I^2]}^{C+1-\alpha} [w_E^2 - t_I^2] f_{E(1:N-1)}(w_E^2) f_I(t_I^2)\, dw_E^2\, dt_I^2 + \int_{C-\alpha}^{C+1-\alpha} \int_{r^2}^{C+1+s} [r^2 - t_I^2] f_{E(1:N-1)}(w_E^2) f_I(t_I^2)\, dw_E^2\, dt_I^2 \Bigg]}_{E[\Pi^2(t_I^2)]} \\
& + \underbrace{\Bigg[\int_{C+s}^{C+1-\alpha} \int_{C+s}^{w_I^2} \int_{t_E^2}^{w_I^2} [w_E^2 - t_E^2] f_{E(1:N-2)}(w_E^2) f_E(t_E^2) f_I(w_I^2)\, dw_E^2\, dt_E^2\, dw_I^2 + \int_{C+s}^{C+1-\alpha} \int_{C+s}^{w_I^2} \int_{w_I^2}^{C+1+s} [w_I^2 - t_E^2] f_{E(1:N-2)}(w_E^2) f_E(t_E^2) f_I(w_I^2)\, dw_E^2\, dt_E^2\, dw_I^2 \Bigg]}_{E[\Pi^2(t_E^2)]}
\end{aligned}$$

B.2 The buyer's expected total cost

The buyer's expected cost at $T = 2$ is as follows.

$$\begin{aligned}
E[\min(\tilde{B}^2(t^2)_{(2:N)},\, r^2)] \;=\; & E[t_{(2:N)}^2 + s|(I,\, E)] \times Pr(I,\, E) + E[r^2|(I,\, r^2)] \times Pr(I,\, r^2) \\
& + E[t_{(2:N)}^2 + s|(E,\, E)] \times Pr(E,\, E) + E[t_{(2:N)}^2|(E,\, I)] \times Pr(E,\, I),
\end{aligned}$$

where $t^2_{(2:N)}$ is the second lowest bid(cost) among N bidders at $T=2$(i.e., an incumbent and $N-1$ entrants) and $Pr(y,z)$ is the probability of (who wins, who defines the payment to the winning supplier) at $T=2$, where I, E, and r^2 denote an incumbent, an entrant, and a reserve price at $T=2$, respectively. The expected cost at $T=2$ is the combination of the expected cost conditional on the pair of (who wins, who defines the payment).

With the given the bidding strategy, the buyer's expected total cost is

$$\begin{aligned}
ETC_{SI} &= E[t^1_{(2:N)}] + E[t^2_I] \\
&\quad + \int_{C+s}^{C+1-\alpha} x \left\{[1-F_I(x)]\, F_{E(2:N-1)}(x)\right\}' dx \\
&\quad + \int_{C+s}^{C+1-\alpha} \left(\int_{C+s}^{x} F_E(y)dy\right) \left\{[1-F_E(x)]\left[1-F_{E(1:N-2)}(x)\right]\right\}' dx \\
&= E[t^1_{(2:N)}] + E[t^2_I] \\
&\quad - \int_{C+s}^{C+1-\alpha} [1-F_I(x)]\, F_{E(2:N-1)}(x)dx \\
&\quad + \int_{C+s}^{C+1-\alpha} [1-F_I(x)]\left[1-F_{E(1:N-2)}(x)\right] F_E(x)dx
\end{aligned}$$

B.3 *proof of* $E[\Pi^2_{SI}(t^2_I)] > E[\Pi^2_{SI}(t^2_E)]$

We show that the supplier t always shades his bid at $T=1$ below his cost t^1 even if he considers the expected profit as an entrant at $T=2$ by which he would inflate his bid.

The expected profit at $T=2$, if supplier t becomes an incumbent, is

$$\begin{aligned}
E[\Pi^2_{SI}(t^2_I)] &= \int_{C-\alpha}^{C+1-\alpha} \int_{\max[C+s,\, t^2_I]}^{C+1-\alpha} [w^2_E - t^2_I] f_{E(1:N-1)}(w^2_E) f_I(t^2_I)\, dw^2_E\, dt^2_I \\
&\quad + \int_{C-\alpha}^{C+1-\alpha} \int_{r^2}^{C+1+s} [r^2 - t^2_I] f_{E(1:N-1)}(w^2_E) f_I(t^2_I)\, dw^2_E\, dt^2_I \\
&= \int_{C-\alpha}^{C+1-\alpha} F_I(x)\, dx - \int_{C+s}^{C+1-\alpha} F_I(x) F_{E(1:N-1)}(x)dx \\
&= \int_{C-\alpha}^{C+s} F_I(x)\, dx + \int_{C+s}^{C+1-\alpha} F_I(x)\, (1-F_E(x))^{N-1}\, dx
\end{aligned}$$

The expected profit at $T = 2$, if supplier t becomes an entrant, is

$$\begin{aligned}
E[\Pi^2_{SI}(t^2_E)] &= \int_{C+s}^{C+1-\alpha} \int_{C+s}^{w^2_I} \int_{t^2_E}^{w^2_I} [w^2_E - t^2_E] f_{E(1:N-2)}(w^2_E) f_E(t^2_E) f_I(w^2_I)\, dw^2_E\, dt^2_E\, dw^2_I \\
&\quad + \int_{C+s}^{C+1-\alpha} \int_{C+s}^{w^2_I} \int_{w^2_I}^{C+1+s} [w^2_I - t^2_E] f_{E(1:N-2)}(w^2_E) f_E(t^2_E) f_I(w^2_I)\, dw^2_E\, dt^2_E\, dw^2_I \\
&= \int_{C+s}^{C+1-\alpha} [1 - F_I(x)] \left(1 - F_E(x)\right)^{N-2} F_E(x) dx
\end{aligned}$$

The difference between the two is as follows.

$$\begin{aligned}
& E[\Pi^2_{SI}(t^2_I)] - E[\Pi^2_{SI}(t^2_E)] \\
= & \int_{C-\alpha}^{C+s} F_I(x)\, dx + \int_{C+s}^{C+1-\alpha} \left(1 - F_E(x)\right)^{N-2} [F_I(x) - F_E(x)]\, dx \\
> & \ 0 \quad (\because \int_{C-\alpha}^{C+s} F_I(x)\, dx > 0, \ \& \ F_E(x) \prec_{FSD} F_I(x))
\end{aligned}$$

The last integral is greater than 0 because $F_E(x)$ first degree stochastically dominates ($\prec_{FSD}$) $F_I(x)$ ($F_I(x)$ and $F_E(x)$ are same distribution with different range, but same intervals, i.e., the lowest value of $F_I(x) <$ that of $F_E(x)$ (this result mirrors to that of Grimm[15]).

APPENDIX C

BIDDING STRATEGIES AND THE OPTIMAL $R^*(T)$ UNDER EPC (IN CHAPTER 4: LI1)

C.1 The equilibrium bidding strategy of a bidder t

A supplier t has three possible positive payoffs streams : when he (a) wins at $T = 1$ and (finds it profitable to) accepts $R(\tau)$ at $T = 2$, (b) wins at $T = 1$ and rejects $R(\tau)$ at $T = 2$, and (c) loses at $T = 1$ and wins at $T = 2$. Note that the payment at $T = 2$, $R(\tau)$ is determined by the bidder t's reported type τ. Here, we use α_t as the maximum possible cost reduction(i.e., α_t is not necessarily same for all type t). For simple notation, we drop superscript 1 from a supplier's cost at $T = 1$.

(<u>case1</u>) $\tau < t$

We consider two cases where (i) $t - \alpha_t < R(t) \leq t$ and (ii) $R(t) = t - \alpha_t$.

(i) $t - \alpha_t < R(t) \leq t$

By monotonicity of R, $R(\tau) < R(t) \leq t$. By continuity of R, there exists $\epsilon > 0$, s.t. $t - \alpha_t \leq R(t - \epsilon) = R(\tau)$. Therefore, $t - \alpha_t \leq R(\tau) < R(t) \leq t$. The supplier t's payoff is,

$$\begin{aligned}
\Pi(t,\tau) \quad = \quad & \int_{t-\alpha_t}^{R(\tau)} \int_{\tau}^{C+1} [B(w) + R(\tau) - t - t_I^2]\, f_{(1:N-1)}(w) f_I(t_I^2)\, dw\, dt_I^2 \\
& + \int_{R(\tau)}^{t} \int_{\tau}^{C+1} [B(w) - t]\, f_{(1:N-1)}(w) f_I(t_I^2)\, dw\, dt_I^2 \\
& + \int_{C}^{\tau} \int_{R(w)}^{w} f_I(w_I^2) f_{(1:N-1)}(w)\, dw_I^2\, dw \\
& \times \int_{C}^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2
\end{aligned} \tag{C.1}$$

where t_S^T denotes the bidder t at time T as the status of S (i.e., I if he is an incumbent, E if he is an entrant), w^1 is the lowest bidder among $N-1$ bidders excluding bidder t at $T = 1$, w_S^2 denotes the bidder $w(\neq t)$ at $T = 2$ as the status of S (i.e., I if he is an incumbent, E if he is an entrant). The buyer's choice of $R(t)$ for each $t \in [C, C + 1]$ is restricted to

$R(t) \in [t-\alpha_t, t]$, for each type (i.e., $0 < t - R(\tau) < \alpha_t$ and $0 < w - R(w) < \alpha_w)$[1].

After solving the first order condition, the equilibrium bidding strategy is,

$$\begin{aligned} B^1_{EPC}(t) &= t \\ &- \int_{t-\alpha_t}^{R(t)} \left(R(t) - t_I^2 - \frac{R'(t)h(t)}{(N-1)} \right) f_I(t_I^2)\, dt_I^2 \\ &+ \int_{R(t)}^{t} f_I(w_I^2)\, dw_I^2 \int_C^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2 \end{aligned} \tag{C.2}$$

(ii) $R(t) = t - \alpha_t$

In this case, $R(\tau) < t - \alpha_t = R(t)$. Thus, the incumbent will always reject $R(\tau)$. The payoff function is reduced to as follows.

$$\begin{aligned} \Pi(t,\tau) &= \int_{t-\alpha_t}^{t} \int_{\tau}^{C+1} [B(w) - t]\, f_{(1:N-1)}(w) f_I(t_I^2)\, dw\, dt_I^2 \\ &+ \int_C^{\tau} \int_{R(w)}^{w} f_I(w_I^2) f_{(1:N-1)}(w)\, dw_I^2\, dw \\ &\times \int_C^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2 \end{aligned} \tag{C.3}$$

After solving the first order condition, the bidding strategy is,

$$B(t) = t + \int_C^{C+1} \int_{t_E^2}^{C+1} (w_E^2 - t_E^2) f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2,$$

which is same as in equation (E.2) if we plug $R(t) = t - \alpha_t$ into the equation.

(<u>case2</u>) $\tau > t$

We have two cases where (iii) $t - \alpha_t \leq R(t) < t$ and (iv) $R(t) = t$.

(iii) $t - \alpha_t \leq R(t) < t$

By monotonicity and continuity of R, $t - \alpha_t \leq R(t) < R(\tau) \leq t$, in which the supplier t has the same payoff as in (case 1) (i) - equation (E.1), thus the bidding strategy is the same as in equation (E.2).

(iv) $R(t) = t$

We have $R(t) = t < R(\tau)$. The incumbent always accepts R at $T = 2$. The payoff

[1]Note that the buyer selects the schedule of $R(t)$ based on the type (cost) which is no greater than the type itself (i.e., $R(t) \leq t$). The buyer has indifference between to choose $R(t) = t - \alpha_t$ and $R(t) < t - \alpha_t$ which gives the lower bound of $R(t)$.

function is, then,

$$
\begin{aligned}
\Pi(t,\tau) &= \int_{t-\alpha_t}^{R(\tau)} \int_{\tau}^{C+1} [B(w) + R(\tau) - t - t_I^2]\, f_{(1:N-1)}(w) f_I(t_I^2)\, dw\, dt_I^2 \\
&+ \int_C^{\tau} \int_{R(w)}^{w} f_I(w_I^2) f_{(1:N-1)}(w)\, dw_I^2\, dw \\
&\times \int_C^{C+1} \int_{t_E^2}^{C+1} [w_E^2 - t_E^2] f_{(1:N-2)}(w_E^2) f(t_E^2)\, dw_E^2\, dt_E^2
\end{aligned}
$$

The bidding strategy is,

$$
B(t) = \int_{t-\alpha_t}^{t} t_I^2 f_I(t_I^2)\, dt_I^2 + \frac{1-F(t)}{(N-1)f(t)}
$$

This is same as in equation (E.2) if we plug $R(t) = t$ into the equation.

C.2 The optimal $R^*(t) = kt$

The buyer's expected total cost under EPC is,

$$
\begin{aligned}
ETC_{EPC}(R(t)) &\equiv ETC_{EPC}(k) \quad (\leftarrow R(t) = kt) \\
&= E[B_{EPC}^1(t)_{(2:N)}] + E[P_1 \times R(t)_{(1:N)} + P_0 \times (B_{EPC}^2(t)_{(2:N-1)} + s)] \\
&= \int_C^{C+1} B_{EPC(2:N)}^1(t) f_{(2:N)}(t) dt \\
&\quad + \int_C^{C+1} \left(P_1 R(t) + P_0 E[B_{EPC}^2(t)_{(2:N-1)} + s]\right) f_{(1:N)}(t) dt \\
&= \int_C^{C+1} \left(t - E[\Pi_I^2] + E[\Pi_E^2]\right) f_{(2:N)}(t) dt \quad \circledast \\
&\quad + \int_C^{C+1} F_I(R(t)) R(t) f_{(1:N)}(t) dt \\
&\quad + \int_C^{C+1} [1 - F_I(R(t))]\, E[B_{EPC}^2(t)_{(2:N-1)} + s] f_{(1:N)}(t) dt \\
&= \int_C^{C+1} \left(t - E[\Pi_I^2(k)] + E[\Pi_E^2(k)]\right) f_{(2:N)}(t) dt \\
&\quad + \int_C^{C+1} F_I(kt) kt f_{(1:N)}(t) dt \\
&\quad + \int_C^{C+1} [1 - F_I(kt)]\, E[t_{(2:N-1)} + s] f_{(1:N)}(t) dt
\end{aligned}
$$

$\circledast$ Under the uniform F and G, the equilibrium bidding strategy of supplier t at $T = 1$ is,

$$
B_{EPC}^1(t) = t - \underbrace{Pr(t_I^2 \le R(t)) \left[R(t) - E[t_I^2 | t_I^2 \le R(t)] - \frac{R'(t)h(t)}{(N-1)}\right]}_{E[\Pi_I^2]} + \underbrace{\frac{t - R(t)}{\alpha N(N-1)}}_{E[\Pi_E^2]} \quad \forall\, t
$$

where $h(t) = \frac{1-F(t)}{f(t)} = 1 + C - t$, $Pr(t_I^2 \le R(t)) = F_I(R(t))$ and $E[t_I^2 | t_I^2 \le R(t)] = \frac{t-\alpha+R(t)}{2}$.

Under the uniform F and G, $ETC_{EPC}(k)$ is a quadratic function of k. We will find the optimal $k = 1$ by showing that $ETC_{EPC}(k)$ is decreasing in k under the support of $[1 - \frac{\alpha}{C+1}, 1]$ (in our assumption, $R(t) = kt \in [t - \alpha, t] \Rightarrow k \in [\max(1 - \frac{\alpha}{t}, \forall t), 1]$). Thus, we show that (1) $ETC_{EPC}(k)$ is a convex function of k, and (2) $\frac{\partial ETC_{EPC}(k)}{\partial k} |_{k=1} < 0$.

$$
\begin{aligned}
\frac{\partial ETC_{EPC}(k)}{\partial k} &= -\int_C^{C+1} \frac{\partial E[\Pi_I^2(k)]}{\partial k} f_{(2:N)}(t)dt \\
&\quad + \int_C^{C+1} \frac{\partial E[\Pi_E^2(k)]}{\partial k} f_{(2:N)}(t)dt \\
&\quad + \int_C^{C+1} [F_I(kt) + f_I(kt)kt]\, t f_{(1:N)}(t)dt \\
&\quad - \int_C^{C+1} E[t_{(2:N-1)} + s] f_I(kt) t f_{(1:N)}(t)dt \\
&= -\int_C^{C+1} \left[\frac{2t}{\alpha}\left(\frac{t}{2} - \frac{h(t)}{N-1}\right)k - \frac{t-\alpha}{\alpha}\left(t - \frac{h(t)}{N-1}\right)\right] f_{(2:N)}(t)dt \\
&\quad - \int_C^{C+1} \frac{t}{\alpha N(N-1)} f_{(2:N)}(t)dt \\
&\quad + \int_C^{C+1} \left[\frac{\alpha - t + kt}{\alpha} + \frac{kt}{\alpha}\right] t f_{(1:N)}(t)dt \\
&\quad - \int_C^{C+1} E[t_{(2:N-1)} + s] \frac{t}{\alpha} f_{(1:N)}(t)dt \\
&= \left(\int_C^{C+1} \left[-\frac{2t}{\alpha}\left(\frac{t}{2} - \frac{h(t)}{N-1}\right) f_{(2:N)}(t) + \frac{2t}{\alpha} t f_{(1:N)}(t)\right] dt\right) k \\
&\quad + \int_C^{C+1} \frac{t-\alpha}{\alpha}\left(t - \frac{h(t)}{N-1}\right) f_{(2:N)}(t)dt - \frac{1}{\alpha N(N-1)} \int_C^{C+1} t f_{(2:N)}(t)dt \\
&\quad + \int_C^{C+1} \frac{\alpha - t}{\alpha} t f_{(1:N)}(t)dt - \frac{E[t_{(2:N-1)} + s]}{\alpha} \int_C^{C+1} t f_{(1:N)}(t)dt \\
&= \frac{1}{\alpha}\left((1+C)^2 - 2(1+C)\frac{N}{N+1} + \frac{N}{N+2}\right) k \\
&\quad + \frac{1}{\alpha}\left(\frac{1+C}{N+1} - \frac{N}{(N+1)(N+2)} - \frac{1}{N(N-1)}\left(\frac{2}{N+1} + C\right)\right. \\
&\qquad \left. - \left(\frac{1}{N+1} + C\right)\left(\frac{2}{N} + C + s\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 ETC_{EPC}(k)}{\partial k^2} &= \frac{1}{\alpha}\left((1+C)^2 - 2(1+C)\frac{N}{N+1} + \frac{N}{N+2}\right) \\
&= \frac{1}{\alpha}\left((1+C)^2 - 2(1+C) + \frac{2(1+C)}{N+1} + \frac{N}{N+2}\right) \\
&= \frac{1}{\alpha}\left(C^2 - 1 + \frac{2C}{N+1} + \frac{2}{N+1} + \frac{N}{N+2}\right) \\
&= \frac{1}{\alpha}\left(C^2 + \frac{2C}{N+1} + \frac{2}{(N+1)(N+2)}\right) \\
&> 0
\end{aligned}
$$

Thus, $ETC_{EPC}(k)$ is a convex function of k. Now we check the sign of $\frac{\partial ETC_{EPC}(k)}{\partial k}\ |_{k=1}$.

$$
\begin{aligned}
\frac{\partial ETC_{EPC}(k)}{\partial k}\ |_{k=1} &= \frac{1}{\alpha}\left(C^2 + \frac{2C}{N+1} + \frac{2}{(N+1)(N+2)}\right) \\
&\quad + \frac{1}{\alpha}\left(\frac{1+C}{N+1} - \frac{N}{(N+1)(N+2)} - \frac{1}{N(N-1)}\left(\frac{2}{N+1} + C\right)\right. \\
&\quad \left. - \left(\frac{1}{N+1} + C\right)\left(\frac{2}{N} + C + s\right)\right) \\
&= \frac{1}{\alpha}\left(C^2 + \frac{2C}{N+1} + \frac{2}{(N+1)(N+2)} + \frac{1}{N+1} + \frac{C}{N+1} - \frac{N}{(N+1)(N+2)}\right. \\
&\quad - \frac{2}{N(N-1)(N+1)} - \frac{C}{N(N-1)} - \frac{2}{N(N+1)} - \frac{C}{N+1} - \frac{s}{N+1} \\
&\quad \left. - \frac{2C}{N} - C^2 - Cs\right) \\
&= \frac{1}{\alpha}\left(\frac{\Psi}{(N-1)N(N+1)(N+2)} - \frac{s}{N+1} - Cs\right) \\
&\quad \text{where} \quad \Psi = (2-3C)N^2 - (8+5C)N + 2C
\end{aligned}
$$

$\frac{\partial^2 \Psi}{\partial N^2} = 2(2-3C) < 0$ ($\because C \geq 1$), and $N^* = \frac{8+5C}{2(2-3C)} < 0$, where N^* maximizes Ψ. Therefore, for $N \geq 3$, Ψ is decreasing in N. Since $\Psi = 9(2-3C) - 3(8+5C) + 2C = -(6+40C) < 0$ for $N = 3$, $\Psi < 0$ with any $N > 3$. Therefore, $\frac{\partial ETC_{EPC}(k)}{\partial k}\ |_{k=1} < 0$.

In summary, we showed that $ETC_{EPC}(k)$ is decreasing in $k \in [1 - \frac{\alpha}{C+1}, 1]$ by showing that (1) $ETC_{EPC}(k)$ is convex and (2) $\frac{\partial ETC_{EPC}(k)}{\partial k}\ |_{k=1} < 0$. Therefore, the optimal k which minimizing $ETC_{EPC}(k)$ is 1.

C.2.1 The optimal $R^*(t) = kt$ with proportional α

The procedure is the same as the same α.

$$
\begin{aligned}
\frac{\partial ETC_{EPC}(k)}{\partial k} &= \frac{1}{\alpha}\left(\frac{1}{C}+\frac{1}{N+1}\right)k \\
&+\frac{1}{\alpha}\left(-\frac{1}{N(N-1)}-\frac{2}{N}-C-s\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 ETC_{EPC}(k)}{\partial k^2} &= \frac{1}{\alpha}\left(\frac{1}{C}+\frac{1}{N+1}\right) > 0 \\
\frac{\partial ETC_{EPC}(k)}{\partial k}\;|_{k=1} &= -\frac{1}{\alpha}\left(\frac{N+2}{N(N+1)}+\frac{1}{N(N-1)}+s\right) < 0
\end{aligned}
$$

Thus, the quadratic function of $ETC_{EPC}(k)$ is decreasing in $k \leq 1$.

C.3 sensitivity of k

We check the sensitivity of the choice of k. That is, how bad the buyer will be if she can not choose k optimally.

$$
\frac{\partial ETC_{EPC}(k)}{\partial k}\;|_{k=1} = \frac{1}{\alpha}\left(\frac{(2-3C)N^2-(8+5C)N+2C}{(N-1)N(N+1)(N+2)}-\frac{s}{N+1}-Cs\right)
$$

If α increases, $\frac{\partial ETC_{EPC}(k)}{\partial k}\;|_{k=1}$ decreases. That is, the change of $ETC_{EPC}(k)$ (i.e., $ETC_{EPC}(k+\epsilon) - ETC_{EPC}(k)$) for a unit change of k (i.e., ϵ) becomes smaller. When N increases, $\frac{\partial ETC_{EPC}(k)}{\partial k}\;|_{k=1}$ also decreases because the two negative terms in the parenthesis in equation (C.3) decreases. When s increases, the last two terms in the parenthesis in equation (C.3) increases, and hence $\frac{\partial ETC_{EPC}(k)}{\partial k}\;|_{k=1}$ increases. In summary, the buyer's costs are most sensitive to a change in k when α and N are small and s is large.

C.4 The buyer's expected total cost

Given the optimal $R^*(t) = t$, we have the following buyer's expected total cost.

$$
\begin{aligned}
ETC_{EPC}(R^*(t)) &= E[B^1_{EPC}(t)_{(2:N)}] + E[P_1 \times R(t)_{(1:N)} + P_0 \times (B^2_{EPC}(t)_{(2:N-1)} + s)] \\
&= \int_C^{C+1} \left(E[t_I^2] + \frac{1 - F(t)}{f(t)(N-1)} \right) f_{(2:N)}(t)\, dt + \int_C^{C+1} t f_{(1:N)}(t)\, dt \\
&= E[t^2_{I(2:N)}] + \int_C^{C+1} \left(\frac{F(t)}{f(t)} + t \right) f_{(1:N)}(t)\, dt \\
&= E[t^2_{I(2:N)}] + E[t_{(2:N)}] \\
&= 2E[t_{(2:N)}] - \frac{\alpha}{2}
\end{aligned}
$$

APPENDIX D

THE PAYOFF FUNCTION OF A BIDDER UNDER SI (IN CHAPTER 4 : LI1)

The expected profit of bidder t is,

$$
\begin{aligned}
\Pi(t,\tau) = & \int_{t-\alpha_t}^{\min[t,\,\tau]} \int_{\max\,[C+s,\,t_I^2]}^{\tau} \int_{\tau}^{C+1} && [B(w) + w_E^2 - t - t_I^2]\, f_{(1:N-1)}(w) \times \\
& && f_{(1:N-1)}(w_E^2) f_I(t_I^2)\, dw\, dw_E^2\, dt_I^2 \\
& + \int_{t-\alpha_t}^{t} \int_{\max[\tau,\,C+s]}^{C+1+s} \int_{\tau}^{C+1} && [B(w) + \tau - t - t_I^2]\, f_{(1:N-1)}(w) \times \\
& && f_{(1:N-1)}(w_E^2) f_I(t_I^2)\, dw\, dw_E^2\, dt_I^2 \\
& + \int_{\max[C+s,\,t-\alpha_t]}^{t} \int_{C+s}^{\min\,[t_I^2,\,\tau]} \int_{\tau}^{C+1} && [B(w) - t]\, f_{(1:N-1)}(w) \times \\
& && f_{(1:N-1)}(w_E^2) f_I(t_I^2)\, dw\, dw_E^2\, dt_I^2 \\
& + \int_{C+s}^{\tau} \int_{\max\,[C+s,\,w-\alpha_w]}^{w} \int_{C+s}^{w_I^2} \int_{t_E}^{w_I^2} && [w_E^2 - t_E^2]\, f_{(1:N-2)}(w_E^2) f(t_E^2) \times \\
& && f_I(w_I^2) f_{(1:N-1)}(w)\, dw_E^2\, dt_E^2\, dw_I^2\, dw \\
& + \int_{C+s}^{\tau} \int_{\max\,[C+s,\,w-\alpha_w]}^{w} \int_{C+s}^{w_I^2} \int_{w_I^2}^{C+1+s} && [w_I^2 - t_E^2]\, f_{(1:N-2)}(w_E^2) f(t_E^2) \times \\
& && f_I(w_I^2) f_{(1:N-1)}(w)\, dw_E^2\, dt_E^2\, dw_I^2\, dw
\end{aligned}
$$

APPENDIX E

BIDDING STRATEGIES AND THE OPTIMAL $R^*(T)$ UNDER EPC (IN CHAPTER 5 : LI2)

In this appendix, we analyze the symmetric bidding strategy of a bidder of type t in general case(i.e., α_t differs for different t) under LI2. The procedure to obtain the bid under EPC is very similar to the case under LI1 (see appendix C). Recall that the magnitude of the learning effect cost is uncertain, while the operating cost is give by t^1 at $T = 2$. Thus, instead of $F_I(t_I^2)$ as the incumbent's cost distribution at $T = 2$, we use $G(\tilde{\alpha}_t)$ as the distribution of the cost reduction : $G(0) = 0$, and $G(\alpha_t) = 1$, with corresponding probability g (We can easily interchange F_I and G).

E.1 The equilibrium bidding strategy of a bidder t

A supplier t has three possible positive payoffs streams : when he (a) wins at $T = 1$ and (finds it profitable to) accepts $R(\tau)$ at $T = 2$, (b) wins at $T = 1$ and rejects $R(\tau)$ at $T = 2$, and (c) loses at $T = 1$ and wins at $T = 2$. Note that the payment at $T = 2$ $R(\tau)$ is determined by the bidder t's reported type τ.

(<u>case1</u>) $\tau < t$

We consider two cases where (i) $t - \alpha_t < R(t) \le t$ and (ii) $R(t) = t - \alpha_t$.

(i) $t - \alpha_t < R(t) \le t$

The supplier t's payoff is,

$$\begin{aligned}
\Pi(t,\tau) \quad = \quad & \int_{t-R(t)}^{\alpha_t} \int_{\tau}^{C+1} [B(w) + R(t) - 2t + \tilde{\alpha_t}]\, f_{(1:N-1)}(w) g(\tilde{\alpha_t})\, dw\, d\tilde{\alpha}_t \\
& + \int_{0}^{t-R(t)} \int_{\tau}^{C+1} [B(w) - t]\, f_{(1:N-1)}(w) g(\tilde{\alpha_t})\, dw\, d\tilde{\alpha}_t \\
& + \int_{C}^{\tau} \int_{0}^{w_I - R(w_I)} g(\tilde{\alpha_{w_I}}) f_{(1:N-1)}(w_I)\, d\tilde{\alpha}_{w_I}\, dw_I \\
& \times \int_{C}^{C+1} \int_{t_E}^{C+1} [w_E - t_E] f_{(1:N-2)}(w_E) f(t_E)\, dw_E\, dt_E
\end{aligned} \tag{E.1}$$

Subscriptions, I and E, represent the status of the bidder at $T = 2$ (Incumbent and Entrant). For example, w_I is the incumbent (other than type t) , w_E is the entrant other than t, and t_E is the type t who is entrant at $T = 2$. $\tilde{\alpha}_{w_I}$ is the realized cost reduction for the type w_I. The buyer's choice of $R(t)$ for each $t \in [C, C+1]$ is restricted to $R(t) \in [t - \alpha_t, t]$, for each type (i.e., $0 < t - R(\tau) < \alpha_t$ and $0 < w_I - R(w_I) < \alpha_{w_I}$)[1].

After solving the first order condition, the equilibrium bidding strategy is,

$$\begin{aligned} B^1_{EPC}(t) &= t \\ &- \int_{t-R(t)}^{\alpha_t} \left(R(t) - (t - \tilde{\alpha_t}) - \frac{R'(t)h(t)}{(N-1)} \right) g(\tilde{\alpha_t})\, d\tilde{\alpha_t} \\ &+ \int_0^{t-R(t)} g(\tilde{\alpha}_{w_I})\, d\tilde{\alpha}_{w_I} \int_C^{C+1} \int_{t_E}^{C+1} [w_E - t_E] f_{(1:N-2)}(w_E) f(t_E)\, dw_E\, dt_E \end{aligned} \tag{E.2}$$

(ii) $R(t) = t - \alpha_t$

In this case, the incumbent will always reject $R(t)$. The payoff function is reduced to as follows.

$$\begin{aligned} \Pi(t,\tau) &= \int_0^{\alpha_t} \int_\tau^{C+1} [B(w) - t]\, f_{(1:N-1)}(w) g(\tilde{\alpha_t})\, dw\, d\tilde{\alpha_t} \\ &+ \int_C^{\tau} \int_0^{w_I - R(w_I)} g(\tilde{\alpha_{w_I}}) f_{(1:N-1)}(w_I)\, d\tilde{\alpha}_{w_I}\, dw_I \\ &\times \int_C^{C+1} \int_{t_E}^{C+1} [w_E - t_E] f_{(1:N-2)}(w_E) f(t_E)\, dw_E\, dt_E \end{aligned} \tag{E.3}$$

After solving the first order condition, the bidding strategy is,

$$B^1_{EPC}(t) = t + \int_C^{C+1} \int_{t_E}^{C+1} (w_E - t_E) f_{(1:N-2)}(w_E) f(t_E)\, dw_E\, dt_E,$$

which is same as in equation (E.2) when $R(t) = t - \alpha_t$.

(case2) $\tau > t$

We have two cases where (iii) $t - \alpha_t \le R(t) < t$ and (iv) $R(t) = t$.

(iii) $t - \alpha_t \le R(t) < t$

The supplier t has the same payoff as in (case 1) (a) - equation (E.1), thus the bidding strategy is as in equation (E.2).

[1]Note that the buyer selects the schedule of $R(t)$ based on the type (cost) which is no greater than the type, itself (i.e., $R(t) \le t$). And the buyer has indifference between to choose $R(t) = t - \alpha_t$ and $R(t) < t - \alpha_t$ which gives the lower bound of $R(t)$.

(iv) $R(t) = t$

The incumbent always accepts R at $T = 2$. The payoff function is, then,

$$\begin{aligned}
\Pi(t,\tau) &= \int_0^{\alpha_t}\int_\tau^{C+1} [B(w) + R(t) - 2t + \tilde{\alpha_t}]\, f_{(1:N-1)}(w) g(\tilde{\alpha_t})\, dw\, d\tilde{\alpha}_t \\
&+ \int_C^\tau \int_0^{w_I - R(w_I)} g(\tilde{\alpha_{w_I}}) f_{(1:N-1)}(w_I)\, d\tilde{\alpha}_{w_I}\, dw_I \\
&\times \int_C^{C+1}\int_{t_E}^{C+1} [w_E - t_E] f_{(1:N-2)}(w_E) f(t_E)\, dw_E\, dt_E
\end{aligned} \tag{E.4}$$

The bidding strategy is,

$$\begin{aligned}
B^1_{EPC}(t) &= 2t - \int_0^{\alpha_t} \tilde{\alpha}_t g(\tilde{\alpha}_t)\, d\tilde{\alpha}_t - R(t) + \frac{\int_t^{C+1} f_{(1:N-1)}(w)\, dw}{f_{(1:N-1)}(t)} R'(t) \qquad \text{(E.5)} \\
&+ \int_0^{t-R(t)} g(\tilde{\alpha}_{w_I})\, d\tilde{\alpha}_{w_I} \int_C^{C+1}\int_{t_E}^{C+1} (w_E - t_E) f_{(1:N-2)}(w_E) f(t_E)\, dw_E\, dt_E \\
&= t - \int_0^{\alpha_t} \tilde{\alpha}_t g(\tilde{\alpha}_t)\, d\tilde{\alpha}_t + \frac{1 - F(t)}{(N-1) f(t)}
\end{aligned}$$

The last term (E.5) becomes 0 because $R(t) = t$. And this is same as in equation (E.2) if we substitute $R(t)$ with t.

E.2 The optimal $R^*(t) = kt$

The procedure is very similar to that in appendix C.2. First, we take the first derivative of ETC and then rearrange it as follows.

$$\begin{aligned}
\frac{\partial ETC_{EPC}(k)}{\partial k} &= \left(\int_C^{C+1} \left[-\frac{t^2}{\alpha} f_{(2:N)}(t) + \frac{2t^2}{\alpha} f_{(1:N)}(t) \right] dt \right) k \\
&+ \int_C^{C+1} \frac{t-\alpha}{\alpha} t f_{(2:N)}(t) dt - E[\Pi_E] \int_C^{C+1} \frac{t}{\alpha} f_{(2:N)}(t) dt \\
&+ \int_C^{C+1} \frac{\alpha - t}{\alpha} t f_{(1:N)}(t) dt - \frac{E[t_{(2:N-1)} + s]}{\alpha} \int_C^{C+1} t f_{(1:N)}(t) dt
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 ETC_{EPC}(k)}{\partial k^2} &= \int_C^{C+1} \left[-\frac{t^2}{\alpha} f_{(2:N)}(t) + \frac{2t^2}{\alpha} f_{(1:N)}(t) \right] dt \\
&= \frac{1}{\alpha} \left(C^2 + C + \frac{N(N+1)}{(N+2)} \right) \\
&> 0
\end{aligned}$$

Thus, $ETC_{EPC}(k)$ is a convex function of k. Now we check the sign of $\frac{\partial ETC_{EPC}(k)}{\partial k}\,|_{k=1}$.

$$
\begin{aligned}
\frac{\partial ETC_{EPC}(k)}{\partial k}\mid_{k=1} &= \frac{1}{\alpha}\left(\frac{\Psi}{(N-1)N(N+1)(N+2)} - \frac{s}{N+1} - Cs\right) - \frac{1}{N+1} \\
&\quad \text{where} \quad \Psi = -CN^4 - 3CN^3 + (1-2C)N^2 + (2C-6)N + 4C - 1
\end{aligned}
$$

$\frac{\partial^2\Psi}{\partial N^2} = -6CN(2N+3) < 0$, and $\frac{\partial\Psi}{\partial N}_{N=3} = -201C + 6 < 0 (\because C \geq 1)$. Therefor, $\Psi < 0$ for any $N > 3$.

In summary, we showed that $ETC_{EPC}(k)$ is decreasing in $k \in [1 - \frac{\alpha}{C+1}, 1]$ by showing that (1) $ETC_{EPC}(k)$ is convex and (2) $\frac{\partial ETC_{EPC}(k)}{\partial k} \mid_{k=1} < 0$. Therefore, the optimal k which minimizing $ETC_{EPC}(k)$ is 1.

E.2.1 The optimal $R^*(t) = kt$ with new set of entrants

The bidding strategy of supplier t is,

$$
B^1_{EPC}(t) = t - \int_{t-\alpha_t}^{R(t)} \left(R(t) - t_I^2\right) f_I(t_I^2)\, dt_I^2
$$

Given this, we have decreasing convex function as follows.

$$
\begin{aligned}
\frac{\partial^2 ETC_{EPC}(k)}{\partial k^2} &> 0 (\text{same as the same set of entrants}) \\
\frac{\partial ETC_{EPC}(k)}{\partial k}\mid_{k=1} &= \frac{1}{\alpha}\left(\frac{\Psi}{N(N+1)(N+2)}\right) - \frac{1}{N+1} < 0 \\
&\quad \text{where} \quad \Psi = -\left[CN^2 + CN + 4C + 4\right]
\end{aligned}
$$

E.3 The buyer's expected total cost

Given the optimal $R^*(t) = t$, the bidding strategy and the the buyer's expected total cost as as follows.

$$
\begin{aligned}
B(t) &= t - [R(t) - (t - E[\tilde{\alpha}])] \\
&= t - E[\tilde{\alpha}] \\
ETC_{EPC}(R^*(t)) &= E[B^1_{EPC}(t)_{(2:N)}] + E[P_1 \times R(t)_{(1:N)} + P_0 \times (B^2_{EPC}(t)_{(2:N-1)} + s)] \\
&= \int_C^{C+1} (t - E[\tilde{\alpha}])\, f_{(2:N)}(t)\, dt + \int_C^{C+1} t f_{(1:N)}(t)\, dt \\
&= E[t_{(2:N)}] - E[\tilde{\alpha}] + E[t_{(1:N)}]
\end{aligned}
$$

E.3.1 The buyer's expected total cost with new set of entrants

Given the optimal $R^*(t)$ we have the following $B(t)$ and ETC.

$$\begin{aligned} B(t) &= t - [R(t) - (t - E[\tilde{\alpha}])] = t - E[\tilde{\alpha}] \\ ETC_{EPC}(R^*(t)) &= E[t_{(2:N)}] - E[\tilde{\alpha}] + E[t_{(1:N)}] \end{aligned}$$

APPENDIX F

BIDDING STRATEGIES UNDER SI (IN CHAPTER 5 : LI2)

F.1 The equilibrium bidding strategy of a bidder t

Supplier t has the five possible payoffs during the whole procurement stages. The five possible payoffs cover the cases when supplier t (a) wins in both periods and an entrant defines his payment at $T = 2$, (b) wins in both periods and the buyer pays her reserve price $r^2 = t$, (c) wins only at $T = 1$, (d) wins only at $T = 2$ and an entrant defines his payment and (e) wins only at $T = 2$ and the incumbent defines his payment. Note that the buyer switches her suppliers only when at least one of the entrants' cost is lower than incumbent's cost minus switching cost s. Without loss of generality, we can obtain the payoff of supplier t by shifting the cost range of an entrant by s (we use the same method to obtain the expected total cost at $T = 2$ of the buyer) : we use F_E as the shifted distribution, i.e., $F_E(C + s) = 0$, $F_E(C + 1 + s) = 1$, with corresponding pdf f_E.

(case1) $\tau < t$:

(case1-1) $t < C + s$

When t wins at $T = 1$, he is also wins at $T = 2$ with probability of 1 and the payment is the buyer's reserve price t (all entrants bids $\geq C + s \geq t$). If t loses at $T = 1$, the incumbent w_I will also win at $T = 2$ with probability of 1, and there is no second chance of win for bidder t ($w_I < t < C + s$ at $T = 1$, $t_E \geq C + s$). Therefore bidder t's payoff is as follow.

$$\Pi(t, \tau) = \int_0^{\alpha_t} \int_{\tau}^{C+1} [B(w) + r^2 - 2t + \tilde{\alpha}_t] \, f_{(1:N-1)}(w) g(\tilde{\alpha}_t) \, dw \, d\tilde{\alpha}_t$$

The bidding strategy from the first order condition with replacing r into t as follows.

(Bid 1) When $t < C + s$

$$B_{SI}^1(t) = t - \int_0^{\alpha_t} \tilde{\alpha}_t g(\tilde{\alpha}_t)\, d\tilde{\alpha}_t \tag{F.1}$$

(<u>case1-2</u>) $t - \alpha_t < C + s < t$

First we assume that there exists a certain r s.t $C + s < \tau < t$. The payoff function of the supplier t, when he pretends as type r, is,

$$\begin{aligned}
\Pi(t,\tau) = \int_0^{\alpha_t} \int_{\max[C+s,\, t-\tilde{\alpha}_t]}^{t} \int_{\tau}^{C+1} \quad & [B(w) + w_E - 2t + \tilde{\alpha}_t]\, f_{(1:N-1)}(w) \times \\
& f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t \\
+ \int_0^{\alpha_t} \int_{r^2}^{C+1+s} \int_{\tau}^{C+1} \quad & [B(w) + r^2 - 2t + \tilde{\alpha}_t]\, f_{(1:N-1)}(w) \times \\
& f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t \\
+ \int_0^{t-C-s} \int_{C+s}^{t-\tilde{\alpha}_t} \int_{\tau}^{C+1} \quad & [B(w) - t]\, f_{(1:N-1)}(w) \times \\
& f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t \\
+ \int_{C+s}^{\tau} \int_0^{\min[w_I-C-s,\, \alpha_{w_I}]} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{t_E}^{w_I-\tilde{\alpha}_{w_I}} \quad & [w_E - t_E]\, f_{(1:N-2)}(w_E) f(t_E) \times \qquad \text{(F.2)} \\
& g(\tilde{\alpha}_{w_I}) f_{(1:N-1)}(w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I \\
+ \int_{C+s}^{\tau} \int_0^{\min[w_I-C-s,\, \alpha_{w_I}]} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{w_I-\tilde{\alpha}_{w_I}}^{C+1+s} \quad & [w_I - \tilde{\alpha}_{w_I} - t_E]\, f_{(1:N-2)}(w_E) f(t_E) \times \qquad \text{(F.3)} \\
& g(\tilde{\alpha}_{w_I}) f_{(1:N-1)}(w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I
\end{aligned}$$

Let $\digamma(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I) = [w_E - t_E]\, f_{(1:N-2)}(w_E) f(t_E) g(\tilde{\alpha}_{w_I}) f_{(1:N-1)}(w_I)$ (equation (F.2)) and $\daleth(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I) = [w_I - \tilde{\alpha}_{w_I} - t_E]\, f_{(1:N-2)}(w_E) f(t_E) g(\tilde{\alpha}_{w_I}) f_{(1:N-1)}(w_I)$ (equation (F.3)). Depending on whether $w_I - \alpha_{w_I} < C + s$ or not, the last two payoffs change.

<u>Payoff (F.2)+ payoff (F.3)</u> (note : $w_I \geq C + s$) :

If $w_I - C - s < \alpha_{w_I}$, then $\min(w_I - C - s,\, \alpha_{w_I}) = w_I - C - s (\Rightarrow w_I < C + s + \alpha_{w_I})$

$$\begin{aligned}
& \int_{C+s}^{\min(C+s+\alpha_{w_I},\, \tau)} \int_0^{w_I-C-s} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{t_E}^{w_I-\tilde{\alpha}_{w_I}} \digamma(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I \\
+ & \int_{C+s}^{\min(C+s+\alpha_{w_I},\, \tau)} \int_0^{w_I-C-s} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{w_I-\tilde{\alpha}_{w_I}}^{C+1+s} \daleth(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I
\end{aligned}$$

If $w_I - C - s \geq \alpha_{w_I}$, then $\min(w_I - C - s,\, \alpha_{w_I}) = \alpha_{w_I} (\Rightarrow w_I \geq C + s + \alpha_{w_I})$

$$\int_{C+s+\alpha_{w_I}}^{\max(C+s+\alpha_{w_I},\tau)} \int_0^{\alpha_{w_I}} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{t_E}^{w_I-\tilde{\alpha}_{w_I}} \digamma(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$
$$+ \int_{C+s+\alpha_{w_I}}^{\max(C+s+\alpha_{w_I},\tau)} \int_0^{\alpha_{w_I}} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{w_I-\tilde{\alpha}_{w_I}}^{C+1+s} \urcorner(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$

Hence,

(i) if $C + s < \tau < C + s + \alpha_{w_I}$,

$$\int_{C+s}^{\tau} \int_0^{w_I-C-s} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{t_E}^{w_I-\tilde{\alpha}_{w_I}} \digamma(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$
$$+ \int_{C+s}^{\tau} \int_0^{w_I-C-s} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{w_I-\tilde{\alpha}_{w_I}}^{C+1+s} \urcorner(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$

(ii) If $C + s + \alpha_{w_I} \leq \tau$,

$$\int_{C+s}^{C+s+\alpha_{w_I}} \int_0^{w_I-C-s} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{t_E}^{w_I-\tilde{\alpha}_{w_I}} \digamma(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$
$$+ \int_{C+s+\alpha_{w_I}}^{\tau} \int_0^{\alpha_{w_I}} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{t_E}^{w_I-\tilde{\alpha}_{w_I}} \digamma(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$
$$+ \int_{C+s}^{C+s+\alpha_{w_I}} \int_0^{w_I-C-s} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{w_I-\tilde{\alpha}_{w_I}}^{C+1+s} \urcorner(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$
$$+ \int_{C+s+\alpha_{w_I}}^{\tau} \int_0^{\alpha_{w_I}} \int_{C+s}^{w_I-\tilde{\alpha}_{w_I}} \int_{w_I-\tilde{\alpha}_{w_I}}^{C+1+s} \urcorner(w_E,\, t_E,\, \tilde{\alpha}_{w_I}\, w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$

Combined with the assumption $t < C + s + \alpha_t$, (1) when $\alpha_t \leq \alpha_{w_I}$, then only case (i) holds ($C+s < t < C+s+\alpha_t$), (2) when $\alpha_t > \alpha_{w_I}$, (2-1) with $C+s < t < C+s+\alpha_{w_I}$, case (i) holds, and (2-2) with $C + s + \alpha_{w_I} < t < C + s + \alpha_t$, case (ii) holds. Thus, by solving the first order condition and replacing $\tau = t$ in each case, the equilibrium bidding strategy of supplier t at $T = 1$ is

(Bid 2) When $C+s<t<C+s+\alpha_{w_I}$, $\alpha_t>\alpha_{w_I}$, or $C+s<t<C+s+\alpha_t$, $\alpha_t\le\alpha_{w_I}$

$$\begin{aligned}
B^1_{SI}(t) \quad = \quad & \Big[\int_{t-C-s}^{\alpha_t}\int_{C+s}^{t}(2t-\tilde{\alpha}_t-w_E)f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\,dw_E\,d\tilde{\alpha}_t \\
& + \int_0^{t-C-s}\int_{t-\tilde{\alpha}_t}^{t}(2t-\tilde{\alpha}_t-w_E)f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\,dw_E\,d\tilde{\alpha}_t\Big] \\
& + \int_0^{\alpha_t}\int_t^{C+1+s}(t-\tilde{\alpha}_t)f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\,dw_E\,d\tilde{\alpha}_t \\
& + \int_0^{t-C-s}\int_{C+s}^{t-\tilde{\alpha}_t}tf_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\,dw_E\,d\tilde{\alpha}_t \\
& + \int_0^{\Phi}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{t_E}^{t-\tilde{\alpha}_t}(w_E-t_E)f_{(1:N-2)}(w_E)f(t_E)g(\tilde{\alpha}_{w_I})\,dw_E\,dt_E\,d\tilde{\alpha}_{w_I} \\
& + \int_0^{\Phi}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{t-\tilde{\alpha}_t}^{C+1+s}(t-\tilde{\alpha}_{w_I}-t_E)f_{(1:N-2)}(w_E)f(t_E)g(\tilde{\alpha}_{w_I})\,dw_E\,dt_E\,d\tilde{\alpha}_{w_I} \\
& (\text{where} \quad \Phi=t-C-s)
\end{aligned} \tag{F.4}$$

(Bid 3) When $C+s+\alpha_{w_I}\le t<C+s+\alpha_t$

$$\begin{aligned}
B^1_{SI}(t) \quad = \quad & \Big[\int_{t-C-s}^{\alpha_t}\int_{C+s}^{t}(2t-\tilde{\alpha}_t-w_E)f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\,dw_E\,d\tilde{\alpha}_t \\
& + \int_0^{t-C-s}\int_{t-\tilde{\alpha}_t}^{t}(2t-\tilde{\alpha}_t-w_E)f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\,dw_E\,d\tilde{\alpha}_t\Big] \\
& + \int_0^{\alpha_t}\int_t^{C+1+s}(t-\tilde{\alpha}_t)f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\,dw_E\,d\tilde{\alpha}_t \\
& + \int_0^{t-C-s}\int_{C+s}^{t-\tilde{\alpha}_t}tf(w_E)(1-F(w_E))^{N-2}g(\tilde{\alpha}_t)\,dw_E\,d\tilde{\alpha}_t \\
& + \int_0^{\Phi}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{t_E}^{t-\tilde{\alpha}_t}(w_E-t_E)f_{(1:N-2)}(w_E)f(t_E)g(\tilde{\alpha}_{w_I})\,dw_E\,dt_E\,d\tilde{\alpha}_{w_I} \\
& + \int_0^{\Phi}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{t-\tilde{\alpha}_t}^{C+1+s}(t-\tilde{\alpha}_{w_I}-t_E)f_{(1:N-2)}(w_E)f(t_E)g(\tilde{\alpha}_{w_I})\,dw_E\,dt_E\,d\tilde{\alpha}_{w_I} \\
& (\text{where} \quad \Phi=\alpha_t : \text{The only difference from } B(t) \text{ in (F.4) is } \Phi)
\end{aligned} \tag{F.5}$$

(case1-3) $t-\alpha_t>C+s$

We assume that there exists a certain τ s.t. $C+s+\alpha_t<\tau<t$ The payoff function of

the supplier t, when he pretends as type τ, is,

$$
\begin{aligned}
\Pi(t,\tau) = \int_{0}^{\alpha_t}\int_{t-\tilde{\alpha}_t}^{t}\int_{\tau}^{C+1} \quad & [B(w)+w_E-2t+\tilde{\alpha}_t]\, f_{(1:N-1)}(w)\times \\
& f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t \\
+\int_{0}^{\alpha_t}\int_{r^2}^{C+1+s}\int_{\tau}^{C+1} \quad & [B(w)+r^2-2t+\tilde{\alpha}_t]\, f_{(1:N-1)}(w)\times \\
& f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t \\
+\int_{0}^{\alpha_t}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{\tau}^{C+1} \quad & [B(w)-t]\, f_{(1:N-1)}(w)\times \\
& f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t \\
+\int_{C+s}^{\tau}\int_{0}^{\min(w_I-C-s,\,\alpha_{w_I})}\int_{C+s}^{w_I-\tilde{\alpha}_{w_I}}\int_{t_E}^{w_I-\tilde{\alpha}_{w_I}} \quad & [w_E-t_E]\, f_{(1:N-2)}(w_E)f(t_E)\times \qquad \text{(F.6)} \\
& g(\tilde{\alpha}_{w_I})f_{(1:N-1)}(w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I \\
+\int_{C+s}^{\tau}\int_{0}^{\min(w_I-C-s,\,\alpha_{w_I})}\int_{C+s}^{w_I-\tilde{\alpha}_{w_I}}\int_{w_I-\tilde{\alpha}_{w_I}}^{C+1+s} \quad & [w_I-\tilde{\alpha}_{w_I}-t_E]\, f_{(1:N-2)}(w_E)f(t_E)\times \qquad \text{(F.7)} \\
& g(\tilde{\alpha}_{w_I})f_{(1:N-1)}(w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I
\end{aligned}
$$

Payoff (F.6)+ payoff (F.7) : When $\alpha_t \le \alpha_{w_I}$, with (1) $C+s+\alpha_t < t < C+s+\alpha_{w_I}$, it is same as in case (1-2) (i), and with (2) $t \ge C+s+\alpha_{w_I}$, it is same as in case (1-2) (ii). When $\alpha_t > \alpha_{w_I}$, with $C+s+\alpha_t \le t$, it is same as in case (1-2) (ii).

The equilibrium bidding strategy of supplier t at $T=1$ is

(Bid 4) When $C+s+\alpha_t < t$, $\alpha_t > \alpha_{w_I}$, or when $C+s+\alpha_{w_I} \le t$, $\alpha_t \le \alpha_{w_I}$

$$
\begin{aligned}
B^1_{SI}(t) \quad = \quad & \int_{0}^{\alpha_t}\int_{t-\tilde{\alpha}_t}^{t}(2t-\tilde{\alpha}_t-w_E)f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\
& +\int_{0}^{\alpha_t}\int_{t}^{C+1+s}(t-\tilde{\alpha}_t)f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\
& +\int_{0}^{\alpha_t}\int_{C+s}^{t-\tilde{\alpha}_t} t f_{(1:N-1)}(w_E)g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \qquad \text{(F.8)} \\
& +\int_{0}^{\Phi}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{t_E}^{t-\tilde{\alpha}_t}(w_E-t_E)f_{(1:N-2)}(w_E)f(t_E)g(\tilde{\alpha}_{w_I})\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I} \\
& +\int_{0}^{\Phi}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{t-\tilde{\alpha}_t}^{C+1+s}(t-\tilde{\alpha}_{w_I}-t_E)f_{(1:N-2)}(w_E)f(t_E)g(\tilde{\alpha}_{w_I})\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I} \\
& (\text{where} \quad \Phi = \alpha_t)
\end{aligned}
$$

(Bid 5) When $C+s+\alpha_t < t < C+s+\alpha_{w_I}$

$$
\begin{aligned}
B^1_{SI}(t) \quad = \quad & \int_0^{\alpha_t}\int_{t-\tilde{\alpha}_t}^{t} (2t-\tilde{\alpha}_t-w_E) f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\
& + \int_0^{\alpha_t}\int_{t}^{C+1+s} (t-\tilde{\alpha}_t) f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\
& + \int_0^{\alpha_t}\int_{C+s}^{t-\tilde{\alpha}_t} t f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\
& + \int_0^{\Phi}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{t_E}^{t-\tilde{\alpha}_t} (w_E-t_E) f_{(1:N-2)}(w_E) f(t_E) g(\tilde{\alpha}_{w_I})\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I} \\
& + \int_0^{\Phi}\int_{C+s}^{t-\tilde{\alpha}_t}\int_{t-\tilde{\alpha}_t}^{C+1+s} (t-\tilde{\alpha}_{w_I}-t_E) f_{(1:N-2)}(w_E) f(t_E) g(\tilde{\alpha}_{w_I})\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I} \\
& (\text{where} \quad \Phi = t-C-s) : \text{The only difference from } B(t) \text{ in (F.8) is } \Phi)
\end{aligned} \tag{F.9}
$$

Now we check the boundaries where $t = C+s$ and $t = C+s+\alpha_t$.

(case1-4) $t = C+s$

(case1-5) $t = C+s+\alpha_t$

(case2) $\tau > t$:

(case2-1) $t < C+s$

Under the assumption that there exists a certain τ s.t $C+s < \tau < t$, same payoff function holds as in (case1-1). Thus, the equilibrium bidding strategy of supplier t is **(Bid 1)** as in equation (F.1).

(case2-2) $t-\alpha_t < C+s < t$

The payoff function of the supplier t is,

$$\Pi(t,\tau) = \int_0^{\alpha_t} \int_{\max[C+s,\, t-\tilde{\alpha}_t]}^{t} \int_{\tau}^{C+1} [B(w) + w_E - 2t + \tilde{\alpha}_t]\, f_{(1:N-1)}(w) \times f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t$$
$$+ \int_0^{\alpha_t} \int_{r^2}^{C+1+s} \int_{\tau}^{C+1} [B(w) + r^2 - 2t + \tilde{\alpha}_t]\, f_{(1:N-1)}(w) \times f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t$$
$$+ \int_0^{t-C-s} \int_{C+s}^{t-\tilde{\alpha}_t} \int_{r}^{C+1} [B(w) - t]\, f_{(1:N-1)}(w) \times f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t$$
$$+ \int_{C+s}^{\tau} \int_0^{\min[w_I - C - s,\, \alpha_{w_I}]} \int_{C+s}^{w_I - \tilde{\alpha}_{w_I}} \int_{t_E}^{w_I - \tilde{\alpha}_{w_I}} [w_E - t_E]\, f_{(1:N-2)}(w_E) \times f(t_E) g(\tilde{\alpha}_{w_I}) f_{(1:N-1)}(w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$
$$+ \int_{C+s}^{\tau} \int_0^{\min[w_I - C - s,\, \alpha_{w_I}]} \int_{C+s}^{w_I - \tilde{\alpha}_{w_I}} \int_{w_I - \tilde{\alpha}_{w_I}}^{C+1+s} [w_I - \tilde{\alpha}_{w_I} - t_E]\, f_{(1:N-2)}(w_E) \times f(t_E) g(\tilde{\alpha}_{w_I}) f_{(1:N-1)}(w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$

The equilibrium bid is same as in **(Bid 2)** - (equation (F.4)) and **(Bid 3)** - (equation (F.5)).

(case2-3) $t - \alpha_t > C + s$

The payoff function of the supplier t is,

$$\Pi(t,\tau) = \int_0^{\alpha_t} \int_{t-\tilde{\alpha}_t}^{t} \int_{\tau}^{C+1} [B(w) + w_E - 2t + \tilde{\alpha}_t]\, f_{(1:N-1)}(w) \times f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t$$
$$+ \int_0^{\alpha_t} \int_{r^2}^{C+1+s} \int_{\tau}^{C+1} [B(w) + r^2 - 2t + \tilde{\alpha}_t]\, f_{(1:N-1)}(w) \times f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t$$
$$+ \int_0^{\alpha_t} \int_{C+s}^{t-\tilde{\alpha}_t} \int_{\tau}^{C+1} [B(w) - t]\, f_{(1:N-1)}(w) \times f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw\, dw_E\, d\tilde{\alpha}_t$$
$$+ \int_{C+s}^{\tau} \int_0^{\min(w_I - C - s,\, \alpha_{w_I})} \int_{C+s}^{w_I - \tilde{\alpha}_{w_I}} \int_{t_E}^{w_I - \tilde{\alpha}_{w_I}} [w_E - t_E]\, f_{(1:N-2)}(w_E) f(t_E) \times g(\tilde{\alpha}_{w_I}) f_{(1:N-1)}(w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$
$$+ \int_{C+s}^{\tau} \int_0^{\min(w_I - C - s,\, \alpha_{w_I})} \int_{C+s}^{w_I - \tilde{\alpha}_{w_I}} \int_{w_I - \tilde{\alpha}_{w_I}}^{C+1+s} [w_I - \tilde{\alpha}_{w_I} - t_E]\, f_{(1:N-2)}(w_E) f(t_E) \times g(\tilde{\alpha}_{w_I}) f_{(1:N-1)}(w_I)\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I}\, dw_I$$

The equilibrium bid is same as in **(Bid 4)** - (equation (F.8)) and **(Bid 5)** - (equation

(F.9)).

($\underline{\text{case2-4}}$) $t = C + s$

($\underline{\text{case2-5}}$) $t = C + s + \alpha_t$

In summary, the equilibrium bid of supplier t is

$$B^1_{SI}(t) = \begin{cases} t - \int_0^{\alpha_t} \tilde{\alpha}_t g(\tilde{\alpha}_t)\, d\tilde{\alpha}_t, & \text{when } C \le t < C + s \\ U_1(t) + V_1(t), & \text{when } C + s < t < C + s + \alpha_{w_I},\ \alpha_t > \alpha_{w_I}, \\ & \text{or } C + s < t < C + s + \alpha_t,\ \alpha_t \le \alpha_{w_I} \\ U_1(t) + V_2(t), & \text{when } C + s + \alpha_{w_I} \le t < C + s + \alpha_t \\ U_2(t) + V_1(t), & \text{when } C + s + \alpha_t < t < C + s + \alpha_{w_I} \\ U_2(t) + V_2(t), & \text{when } C + s + \alpha_t < t,\ \alpha_t > \alpha_{w_I}, \\ & \text{or } C + s + \alpha_{w_I} \le t,\ \alpha_t \le \alpha_{w_I}, \end{cases}$$

where

$$\left(\begin{array}{ll} U_1(t) = & \int_{t-C-s}^{\alpha_t} \int_{C+s}^{t} (2t - \tilde{\alpha}_t - w_E) f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\ & + \int_0^{t-C-s} \int_{t-\tilde{\alpha}_t}^{t} (2t - \tilde{\alpha}_t - w_E) f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\ & + \int_0^{\alpha_t} \int_t^{C+1+s} (t - \tilde{\alpha}_t) f_{(1:N-1)} g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\ & + \int_0^{t-C-s} \int_{C+s}^{t-\tilde{\alpha}_t} t f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\ U_2(t) = & \int_0^{\alpha_t} \int_{t-\tilde{\alpha}_t}^{t} (2t - \tilde{\alpha}_t - w_E) f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\ & + \int_0^{\alpha_t} \int_t^{C+1+s} (t - \tilde{\alpha}_t) f_{(1:N-1)} g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\ & + \int_0^{\alpha_t} \int_{C+s}^{t-\tilde{\alpha}_t} t f_{(1:N-1)}(w_E) g(\tilde{\alpha}_t)\, dw_E\, d\tilde{\alpha}_t \\ V_1(t) = & \int_0^{t-C-s} \int_{C+s}^{t-\tilde{\alpha}_t} \int_{t_E}^{t-\tilde{\alpha}_t} (w_E - t_E) f_{(1:N-2)}(w_E) f(t_E) g(\tilde{\alpha}_{w_I})\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I} \\ & + \int_0^{t-C-s} \int_{C+s}^{t-\tilde{\alpha}_t} \int_{t-\tilde{\alpha}_t}^{C+1+s} (t - \tilde{\alpha}_{w_I} - t_E) f_{(1:N-1)}(w_E) f(t_E) g(\tilde{\alpha}_{w_I})\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I} \\ V_2(t) = & \int_0^{\alpha_t} \int_{C+s}^{t-\tilde{\alpha}_{w_I}} \int_{t_E}^{t-\tilde{\alpha}_{w_I}} (w_E - t_E) f_{(1:N-2)}(w_E) f(t_E) g(\tilde{\alpha}_{w_I})\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I} \\ & + \int_0^{\alpha_t} \int_{C+s}^{t-\tilde{\alpha}_t} \int_{t-\tilde{\alpha}_t}^{C+1+s} (t - \tilde{\alpha}_{w_I} - t_E) f_{(1:N-2)}(w_E) f(t_E) g(\tilde{\alpha}_{w_I})\, dw_E\, dt_E\, d\tilde{\alpha}_{w_I} \end{array} \right)$$

F.2 Monotonicity of SI

We show $B(t)$ is strict monotone under the uniform distribution.

Proof. In Uniform distribution, the equilibrium bidding strategy of supplier t is

$$B_{SI}^1(t) = \begin{cases} t - \frac{\alpha}{2} & \text{if } C \le t < C+s \\ t + \frac{(1+C+s-t)^N}{N} - \frac{(C+s-t+\alpha)^2}{2\alpha} \\ -\frac{C+s-t+\alpha}{\alpha N} + \frac{t-C-s-1+(1+C+s-t)^N}{\alpha N(N-1)} & \text{if } C+s \le t < C+s+\alpha \\ t + \frac{(1+C+s-t)^N}{N} \\ -\frac{(1+C+s-t+\alpha)^N-(1+C+s-t)^N}{\alpha N(N-1)} + \frac{1}{N(N-1)} & \text{if } C+s+\alpha \le t \end{cases}$$

When $t < C+s$,

$$\frac{\partial B(t)}{\partial t} = 1 > 0$$

When $C+s \le t < C+s+\alpha$,

$$\begin{aligned} \frac{\partial B(t)}{\partial t} &= 1-(1+C+s-t)^{N-1} + \frac{(C+s-t+\alpha)}{\alpha} + \frac{1}{\alpha N} + \frac{1}{\alpha N(N-1)} - \frac{(1+C+s-t)^{N-1}}{\alpha(N-1)} \\ &= \underbrace{1-(1+C+s-t)^{N-1}}_{\ge 0} + \underbrace{\frac{(C+s-t+\alpha)}{\alpha}}_{>0} + \underbrace{\frac{1-(1+C+s-t)^{N-1}}{\alpha(N-1)}}_{\ge 0} > 0 \end{aligned}$$

When $C+s+\alpha \le t$,

$$\frac{\partial B(t)}{\partial t} = \underbrace{1-(1+C+s-t)^{N-1}}_{>0} + \underbrace{\frac{(1+C+s-t+\alpha)^{N-1}-(1+C+s-t)^{N-1}}{\alpha(N-1)}}_{>0} > 0$$

□

APPENDIX G

PROOF OF STRICT MONOTONICITY OF $B_M^1(T)$ UNDER EPC WHEN THE BUYER USES DISCRIMINATORY $R(T)$ (IN CHAPTER 4 : LI1)

In chapter 4.4.1, we examined the optimal "*discriminatory*" eroding schedule $R(t)$. We prove that there exists a $B(t)$ which is strictly monotonically increasing in t in equilibrium under the following assumptions[1] :

ASSUMPTION G.1. $R(t) \in [t - \alpha_t, t] \ \forall \ t$.

ASSUMPTION G.2. $R(t)$ *is differentiable everywhere and is strictly monotonically increasing in* t.

The first assumption is already mentioned in the main content. The second assumption was not necessary when $R(t) = kt$ (nondiscriminatory). However, for us to solve the optimal discriminatory $R(t)$, we make the assumption G.2. This assumption is a mild one that states that a buyer would never require a lower price from a higher type t; this assumption is particularly reasonable since the suppliers are assumed to have no control on their cost reduction $\tilde{\alpha}_t$, i.e., the cost reduction arises merely out of having supplied the buyer once before and is not a direct result of any cost-saving measures.

For simplicity, we use $B(t)$ instead of $B_M^1(t)$ in this chapter. We proceed by proving that (*Claim 1*) $B(t)$ is strictly monotonic in t, and then (*Claim 2*) $B(t)$ is increasing in t.

G.1 EPC

Proof. We define $\Pi(t, \tau)$ to be the expected profit of type t submitting a bid as type r. We separate $\Pi(t, \tau)$ into three cases; (i) bidder t wins at $T = 1$ and accepts $R(t)$ at $T = 2$; (ii)

[1] Note that the proof here does not guarantee that $B(t)$ in equation (4.1) are always strict increasing in t.

bidder t wins at $T = 1$ and rejects $R(t)$ at $T = 2$ and (iii) bidder t loses at $T = 1$ (details are in the appendix E).

(*Claim 1*) Suppose that $B(t)$ is not strictly monotonic in t. Then, $\exists\ t_1,\ t_2$ s.t., for $t_1 < t_2$, (i.e., $t_2 - t_1 < \epsilon$ where $0 < \epsilon \leq \delta)$[2], $B(t_1) = B(t_2)$. Suppose that bidder t_1 bids as type t_2. Consider case (i): If bidder t_1 wins at $T = 1$ with bid $B(t_1)$, then he will win with a bid of $B(t_2)$ and will be paid $R(t_2)$ at $T = 2$, which is strictly greater than $R(t_1)$, hence bidder t_1 is strictly better off bidding as type t_2. Under case (ii), bidder t_1 rejects $R(t_1)$; however, under assumptions 1 and 2, he will accept $R(t_2)$ with positive probability hence is strictly better off bidding as type t_2. Under case (iii), if bidder t_1 loses at $T = 1$ with bid $B(t_1)$, then he will lose with a bid $B(t_2)$. Since he redraws costs at $T = 2$, his expected profits at $T = 2$ are the same under either bid, making him indifferent to bidding as type t_1 or t_2. When combining all three cases, bidder t_1 is strictly better off bidding as type t_2 and hence $B(t_1)$ could not have been part of an equilibrium bidding strategy.

(*Claim 2*) We assume that $B(t)$ is decreasing in t and consider the expected profits of two types, C and $C + \epsilon$ ($\epsilon > 0$). First, suppose $\Pi(C, C) > \Pi(C + \epsilon, C + \epsilon)$. Since $B(C)$ is the highest bid, bidder C will always lose at $T = 1$. Hence the expected profit can be positive only when he wins at $T = 2$ as an entrant (where his cost is newly drawn), which implies that $\Pi(C, C) = \Pi(C + \epsilon, C)$ (any bidder who submits a bid of $B(C)$ faces the same expected profit in both auctions). Hence, type $C + \epsilon$ is strictly better off changing his bid to $B(C)$ (because, by assumption, $\Pi(C + \epsilon, C + \epsilon) < \Pi(C, C) = \Pi(C + \epsilon, C)$). This is a contradiction and therefore $\Pi(C, C) \leq \Pi(C + \epsilon, C + \epsilon)$.

Second, we argue that $\Pi(C + \epsilon, C + \epsilon) < \Pi(C, C + \epsilon)$. Any type that submits a bid of $B(C + \epsilon)$ has a positive (epsilonic) probability that he will win at $T = 1$. That, in combination that the payment at $T = 2$ is same for any type who bids as type $C + \epsilon$(i.e., $R(C + \epsilon)$), implies that bidder C must have a higher expected profit $T = 2$ when he bids $B(C + \epsilon)$ at $T = 1$, than bidder $C + \epsilon$, i.e., $\Pi(C + \epsilon, C + \epsilon) < \Pi(C, C + \epsilon)$. Combining these two inequalities on expected profit implies that $\Pi(C, C) \leq \Pi(C + \epsilon, C + \epsilon) < \Pi(C, C + \epsilon)$. Therefore, under the assumption that $B(t)$ is strictly decreasing in t, type C would be

[2] We consider local optima and hence local perturbations from t

strictly better off deviating from his true type bid $B(C)$ and bidding $B(C+\epsilon)$. Hence, $B(t)$ must be strictly increasing in equilibrium. $\square$

G.2 BEPC

Notice that we can also simply obtain the bid under the bundled auction coupled with eroding price schedule under LI1 similar to the case under LI2. In appendix H, we derive the bidding strategy under BEPC in LI1. We prove that there also exists a strict monotone $B(t)$ with discriminatory $R(t)$.

Proof. Hence, $\Pi(t,\tau) = E[B_{(1:N-1)}(w) + R(\tau) - 2t + \tilde{\alpha}_t \mid B(\tau) < B_{(1:N-1)}(w)]Pr(B(\tau) < B_{(1:N-1)})$, where $B(w)_{(1:N-1)}$ is the lowest bid among $N-1$ bidders except bidder t.

(*Claim 1*) Suppose that $B(t)$ is not strictly monotonic in t. Then, $\exists\ t_1,\ t_2$ s.t., for $t_1 < t_2$, (i.e., $t_2 - t_1 < \epsilon$ where $0 < \epsilon \leq \delta$) $B(t_1) = B(t_2)$. If bidder t_1 submits a bid as type t_2, $B(t_2)$, then his probability of winning and expected payment at $T=1$ remain the same, whereas the payment at $T=2$ increases, under assumption 1 ($R(t_2) > R(t_1)$). Therefore he is strictly better off choosing $B(t_2)$ ($\Pi(t_1,t_1) < \Pi(t_1,t_2)$). Hence $B(t_1)$ could not have been part of an equilibrium bidding strategy.

(*Claim 2*) Since $B(t)$ is *strictly* monotonic in t, $B(t)$ will be either increasing or decreasing in t. Suppose that $B(t)$ is decreasing in t. We argue that type C (the lowest type) is strictly better off choosing a higher type's bid (i.e., $C+\epsilon$, where $B(C+\epsilon) < B(C)$, $\epsilon > 0$). $\Pi(C,C)$ is 0 because supplier C always loses at $T=1$. If he were to bid $B(C+\epsilon)$, however, he would win with a positive probability at $T=1$. That, in the combination of assumptions 1 and 2, implies that type C would be strictly better off deviating from his true type bid $B(C)$ and bidding $B(C+\epsilon)$ ($\Pi(C,C) < \Pi(C,C+\epsilon)$). Hence, $B(t)$ must be strictly increasing in equilibrium. $\square$

APPENDIX H

THE EQUIVALENCE BETWEEN BUNDLED PROCUREMENT MECHANISMS UNDER NLI AND LI1

In section 5.4.4, we discussed the bundled procurement mechanisms : an auction coupled with the eroding price schedule(BEPC) and a bundle auction(BA) under LI2. In this section, we examine the same bundled procurement mechanisms under other two models, NLI and LI1. Different from LI2, we show that these two mechanisms yield the same outcome to the buyer.

H.1 Bundled procurement mechanisms under NLI

PROPOSITION H.1. *Given a supplier market characterized by (N, α, s), the buyer's expected total cost is independent of the choice of R under BEPC.*

Proof. The supplier t's payoff $\Pi(t, \tau)$, and the bidding strategy by the first order condition are as follows.

$$\begin{aligned}
\Pi(t,\tau) &= \int_{C-\alpha}^{C+1-\alpha} \int_{\tau}^{C+1} [B(w^1) + R - t^1 - t_I^2]\, f_{(1:N-1)}(w^1) f_I(t_I^2)\, dw^1\, dt_I^2 \\
B^1_{BEPC}(t^1) &= t^1 - [R - \underbrace{\int_{C-\alpha}^{C+1-\alpha} t_I^2 f_I(t_I^2)\, dt_I^2}_{E[t_I^2]}], \qquad \forall\ t^1
\end{aligned}$$

Given the bidding strategy, the buyer's expected total cost is as follows.

$$\begin{aligned}
ETC_{BEPC}(R) &= E[B^1_{BEPC}(t^1)_{(2:N)}] + R \\
&= E[t^1_{(2:N)}] - \big[R - E[t_I^2]\big] + R \\
&= E[t^1_{(2:N)}] + E[t_I^2]
\end{aligned}$$

□

PROPOSITION H.2. *The buyer's expected cost is the same under BEPC and BA.*

Proof. The payoff of supplier t in BA and the bidding strategy of him are as follows.

$$\begin{aligned}\Pi(t,\tau) &= \int_{C-\alpha}^{C+1-\alpha}\int_{\tau}^{C+1}[2B(w^1)-t^1-t_I^2]\,f_{(1:N-1)}(w^1)f_I(t_I^2)\,dw^1\,dt_I^2\\ B_{BA}^1(t^1) &= \frac{1}{2}[t+\underbrace{\int_{C-\alpha}^{C+1-\alpha} t_I^2 f_I(t_I^2)\,dt_I^2}_{E[t_I^2]}],\quad \forall\, t^1\end{aligned}$$

The expected total cost of the buyer is as follows.

$$\begin{aligned}ETC_{BA} &= 2*E[B_{BA}^1(t^1)_{(2:N)}]\\ &= E[t_{(2:N)}^1]+E[t_I^2]\end{aligned}$$

□

Thus, the buyer's expected total cost by bundling is the same as her cost under EPC with $R = C + 1 - \alpha$.

H.2 Bundled procurement mechanisms under LI1

PROPOSITION H.3. *Given a supplier market characterized by (N,α,s), the buyer's expected total cost when using a non-discriminatory price schedule, i.e., $R(t) = kt$ is the same for any k under BEPC.*

Proof. The supplier t's payoff and the bidding strategy are as follows.

$$\begin{aligned}\Pi(t,\tau) &= \int_{0}^{\alpha_t}\int_{\tau}^{C+1}[B(w)+R(\tau)-2t+\tilde{\alpha}_t]\,f_{(1:N-1)}(w)g(\tilde{\alpha}_t)\,dw\,d\tilde{\alpha}_t\\ B_{BEPC}^1(t) &= t-\left[R(t)-\left(t-\int_0^{\alpha_t}\tilde{\alpha}_t g(\tilde{\alpha}_t)\,d\tilde{\alpha}_t\right)\right]+\frac{R'(t)h(t)}{(N-1)},\quad \forall\, t\\ & \quad where\ h(t)=\frac{1-F(t)}{f(t)}\end{aligned}$$

The expected total cost of the buyer is as follows.

$$\begin{aligned}ETC_{BEPC} &= E[B_{BEPC}^1(t)_{(2:N)}]+E[R(t)_{(1:N)}]\\ &= \int_C^{C+1}\left((2-k)t-\int_0^{\alpha}xg(x)\,dx+\frac{k}{N-1}\frac{1-F(t)}{f(t)}\right)f_{(2:N)}(t)+ktf_{(1:N)}(t)\,dt\\ &= E[t_{(2:N)}^1]+E[t_{I(2:N)}^2]\end{aligned}$$

$\square$

PROPOSITION H.4. *The buyer's expected cost is the same under BEPC and BA.*

Proof. The payoff of supplier t in BA and the bidding strategy are as follows.

$$\begin{aligned}
\Pi(t,\tau)_{BA} &= \int_0^{\alpha_t}\int_\tau^{C+1} [2B(w) - 2t + \tilde{\alpha}_t]\, f_{(1:N-1)}(w) g(\tilde{\alpha}_t)\, dw\, d\tilde{\alpha}_t \\
B^1_{BA}(t^1) &= t^1 - \frac{1}{2}\int_0^{\alpha_t} \tilde{\alpha}_t\, g(\tilde{\alpha}_t)\, d\tilde{\alpha}_t
\end{aligned}$$

The expected total cost of the buyer is as follow.

$$\begin{aligned}
ETC_{BA} &= E[2B^1_{BA}(t)_{(2:N)}] \\
&= 2E[t_{(2:N)}] - \int_0^{\alpha} xg(x)\, dx \\
&= E[t^1_{(2:N)}] + E[t^2_{I(2:N)}]
\end{aligned}$$

$\square$

Thus, the buyer's expected total cost by bundling is the same as her cost under EPC. These propositions H.3 and H.4 also apply to the supplier's payoff.

PROPOSITION H.5. *Supplier t's expected profit is the same for any k in BEPC and BA and is given by,*

$$\Pi^{kt}_{FE}(t) = \Pi_{BA}(t) = \int_t^{C+1} 2(1 - F(x))^{N-1}\, dx \tag{H.1}$$

Proof. First, we show that the bidder t's expected profits with different ks in FE are same. If we substitute $B^1_{FE}(t)$ into $B(w)$ in bidder t's payoff function, we have the following expected profit of bidder t, $\Pi(t)_{FE}$.

$$\begin{aligned}
\Pi^{kt}_{FE}(t) &= \int_0^{\alpha}\int_t^{C+1} \left(\underbrace{(2-k)w - \int_0^{\alpha} xg(x)\, dx + \frac{k}{N-1}\frac{1-F(w)}{f(w)}}_{B(w)} + kt - 2t + \tilde{\alpha}_t \right) \\
&\qquad \times f_{(1:N-1)}(w) g(\tilde{\alpha}_t)\, dw\, d\tilde{\alpha}_t \\
&= \int_t^{C+1} \left((2-k)(w-t) + \frac{k}{N-1}\frac{1-F(w)}{f(w)} \right) f_{(1:N-1)}(w)\, dw \\
&= (2-k)\left[(C+1-t) - \int_t^{C+1} F_{(1:N-1)}(w)\, dw \right] + k\int_t^{C+1} (1-F(w))^{N-1}\, dw \\
&= \int_t^{C+1} 2(1-F(w))^{N-1}\, dw
\end{aligned}$$

$\Pi^{kt}_{FE}(t)$ does not depend on k. In BA, the bidder t's expected profit, $\Pi_{BA}(t)$, given $B(t) = t - \frac{1}{2}\int_0^\alpha xg(x)\,dx$ is,

$$\begin{aligned}\Pi_{BA}(t) &= \int_0^\alpha \int_t^{C+1} [\underbrace{2w - \int_0^\alpha xg(x)\,dx}_{B(w)} - 2t + \tilde{\alpha}_t]\, f_{(1:N-1)}(w) g(\tilde{\alpha}_t)\, dw\, d\tilde{\alpha}_t \\ &= \int_t^{C+1} 2(1 - F(w))^{N-1}\, dw\end{aligned}$$

$\square$

APPENDIX I

EFFICIENCY OF EACH PROCUREMENT MECHANISM

A procurement mechanism is *ex post* "*efficient*" if the lowest effective supplier wins the contract. That is, if there is the case where the buyer switches her suppliers, the supplier should be the one whose cost plus switching cost is less than the incumbent supplier. In that sense, SI is *always* efficient. The other mechanisms are needed to check the probability that the lowest effective cost supplier becomes the actual supplier at $T = 2$. In this appendix, we derive the probability that EPC is efficient under NLI, LI1 and LI2 settings.

I.1 EPC under NLI

Under NLI, there are two cases in which the lowest effective supplier wins at $T = 2$; (a) the incumbent accepts $R(t_I)$ $(t_I \le R(t_I))$, and his cost is less than the effective cost of any other entrants $(t_I < t_E)$, and (b) the incumbent rejects $R(t_I)$ $(t_I > R(t_I))$, and the effective cost of an entrant is less than the incumbent's cost $(t_I > t_E)$. Note that the effective cost of an entrant is his cost plus switching cost s. The probability of the procurement mechanism being efficient is,

(1) when $\alpha + s \ge 1$,

$$\int_{C-\alpha}^{R^*} f_I(x)\,dx = 1 \quad (\because R^* = C + 1 - \alpha)$$

(2) when $\alpha + s < 1$,

$$\begin{aligned}
& \int_{C-\alpha}^{R^*} \int_{\max(t_I,\, C+s)}^{C+s+1} f_{E(1:N-1)}(t_E) f_I(t_I)\,dt_E\,dt_I \;\; (\leftarrow \text{Incumbent accepts } R^*, \;\&\; t_I < t_E) \\
+ \quad & \int_{R^*}^{C+1-\alpha} \int_{C+s}^{t_I} f_{E(1:N-1)}(t_E) f_I(t_I)\,dt_E\,dt_I \;\; (\leftarrow \text{Incumbent rejects } R^*, \;\&\; t_I > t_E) \\
= \quad & 1 - \left(\int_{C+s}^{R^*} F_{E(1:N-1)}(x) f_I(x)\,dx + \int_{R^*}^{C+1-\alpha} \left[1 - F_{E(1:N-1)}(x)\right] f_I(x)\,dx \right) \\
< \quad & 1
\end{aligned}$$

Table I.1 illustrates the efficiency of EPC.

Table I.1: Efficiency of EPC under NLI(%)

$s = 0.1$						$\alpha = 0.1$					
$\alpha \diagdown N$	3	4	5	10	20	$s \diagdown N$	3	4	5	10	20
0.1	53.1	65.2	73.0	87.0	93.6	0.1	53.1	65.2	73.0	87.0	93.6
0.3	71.2	65.8	73.2	87.0	93.6	0.3	71.2	65.8	73.2	87.0	93.6
0.5	86.1	81.8	78.4	87.1	93.6	0.5	86.1	81.8	78.4	87.1	93.6
0.7	96.3	94.8	93.4	88.9	93.6	0.7	96.3	94.8	93.4	88.9	93.6

I.2 EPC with the non-discriminatory $R(t)$ under LI1 and LI2

Under LI1, $R^*(t^1) = t^1$. That is, the incumbent always supplies the buyer at $T = 2$. To be the incumbent is the efficient supplier, this incumbent's cost should be indeed less than any other (potential) entrants' effective cost : $t_I - \tilde{\alpha}_I < t_E$, where $t_I - \tilde{\alpha}_I$ is the incumbent realized cost at $T = 2$. The probability of the procurement mechanism being efficient is,

When $t_I \leq C + s$,

$$\int_C^{C+s} \int_0^{\alpha} g(\tilde{\alpha}_I) f_{(1:N)}(t_I)\, d\tilde{\alpha}_I\, dt_I$$

When $C + s < t_I \leq C + s + \alpha$,

$$\int_{C+s}^{C+s+\alpha} \int_0^{\alpha} \int_{\max[C+s,\, t_I-\tilde{\alpha}_I]}^{C+1+s} f_{(1:N-1)}(t_E) g(\tilde{\alpha}_I) f_{(1:N)}(t_I)\, dt_E\, d\tilde{\alpha}_I\, dt_I$$

When $C + s + \alpha < t_I \leq C + 1$,

$$\int_{C+s+\alpha}^{C+1} \int_0^{\alpha} \int_{t_I-\tilde{\alpha}_I}^{C+1+s} f_{(1:N-1)}(t_E) g(\tilde{\alpha}_I) f_{(1:N)}(t_I)\, dt_E\, d\tilde{\alpha}_I\, dt_I$$

The summation of all three is as follows.

$$\begin{aligned} & 1 - \int_{C+s}^{C+1} \int_0^{\min[C+s,\, t_I-C-s]} F_{E(1:N-1)}(t_I - \tilde{\alpha}_I) g(\tilde{\alpha}_I) f_{I(1:N)}(t_I)\, d\tilde{\alpha}_I\, dt_I \\ < \quad & 1 \end{aligned}$$

Table I.2 illustrates the efficiency of EPC.

Table I.2: Efficiency of EPC under LI1(%)

$s = 0.1$						$\alpha = 0.1$					
$\alpha \diagdown N$	3	4	5	10	20	$s \diagdown N$	3	4	5	10	20
0.1	77.2	78.7	80.7	89.8	97.3	0.1	77.2	78.7	80.7	89.8	97.3
0.3	84.9	86.9	89.0	95.5	99.0	0.3	91.7	93.9	95.7	99.4	100.0
0.5	89.6	91.3	92.9	97.3	99.4	0.5	97.9	98.9	99.4	100.0	100.0
0.7	92.4	93.8	94.9	98.0	99.6	0.7	99.8	99.9	100.0	100.0	100.0

I.3 EPC with the optimal discriminatory $R(t)$ under LI2 and LI2

Under the optimal discriminatory EPC, there are two cases in which the lowest effective supplier wins at $T = 2$; (a) the incumbent accepts $R(t_I)$ $(I - \tilde{\alpha}_I \leq R(t_I))$, and his cost is less than the effective cost of any other entrants $(I - \tilde{\alpha}_I < t_E + s)$, and (b) the incumbent rejects $R(t_I)$ $(t_I - \tilde{\alpha}_I > R(t_I))$, and the effective cost of an entrant is less than the incumbent's cost $(t_I - \tilde{\alpha}_I > t_E + s)$. The probability of the procurement mechanism being efficient is,

When $t_I \leq C + s$,

$$\int_C^{C+s} \int_{t_I - R(t_I)}^{\alpha} g(\tilde{\alpha}_I) f_{(1:N)}(t_I)\, d\tilde{\alpha}_I\, dt_I$$

When $C + s < t_I \leq C + s + \alpha$,

$$\begin{aligned} &\int_{C+s}^{C+s+\alpha} \int_{t_I - R(t_I)}^{\alpha} \int_{\max[C+s,\, t_I - \tilde{\alpha}_I]}^{C+1+s} f_{(1:N-1)}(t_E) g(\tilde{\alpha}_I) f_{(1:N)}(t_I)\, dt_E\, d\tilde{\alpha}_I\, dt_I \\ + &\int_{C+s}^{C+s+\alpha} \int_0^{\min[t_I - R(t_I),\, t_I - C - s]} \int_{C+s}^{t_I - \tilde{\alpha}_I} f_{(1:N-1)}(t_E) g(\tilde{\alpha}_I) f_{(1:N)}(t_I)\, dt_E\, d\tilde{\alpha}_I\, dt_I \end{aligned}$$

When $C + s + \alpha < t_I \leq C + 1$,

$$\begin{aligned} &\int_{C+s+\alpha}^{C+1} \int_{t_I - R(t_I)}^{\alpha} \int_{t_I - \tilde{\alpha}_I}^{C+1+s} f_{(1:N-1)}(t_E) g(\tilde{\alpha}_I) f_{(1:N)}(t_I)\, dt_E\, d\tilde{\alpha}_I\, dt_I \\ + &\int_{C+s+\alpha}^{C+1} \int_0^{t_I - R(t_I)} \int_{C+s}^{t_I - \tilde{\alpha}_I} f_{(1:N-1)}(t_E) g(\tilde{\alpha}_I) f_{(1:N)}(t_I)\, dt_E\, d\tilde{\alpha}_I\, dt_I \end{aligned}$$

APPENDIX J

THE OPTIMAL RESERVE PRICES (IN CHAPTER 6)

J.1 Supplier's expected profit at $T = 2$

Supplier t's expected profits at $T = 2$ as an incumbent ($E[\Pi^2(t_I^2)]$) and an entrant ($E[\Pi^2(t_E^2)]$), given the buyer's reserve prices r_I^2, r_E^2 are as follows.

(<u>case1</u>) $r_I \leq r_E$: (we drop superscript of 2)

$$\begin{aligned}
E[\Pi^2(t_I)] &= \int_{C-\alpha}^{r_I} \int_{\max[C+s,\,t_I]}^{r_I} [w_E - t_I]\; f_{E(1:N-1)}(w_E) f_I(t_I)\, dw_E\, dt_I \\
&\quad + \int_{C-\alpha}^{r_I} \int_{\max[r_I, C+s]}^{C+s+1} [r_I - t_I]\; f_{E(1:N-1)}(w_E) f_I(t_I)\, dw_E\, dt_I \\
E[\Pi^2(t_E)] &= \int_{C+s}^{r_I} \int_{t_I}^{C+s+1} \int_{C+s}^{w_I} [w_I - t_E]\, f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I)\, dt_E\, dw_E\, dw_I \\
&\quad + \int_{r_I}^{C+1-\alpha} \int_{r_E}^{C+s+1} \int_{C+s}^{r_E} [r_E - t_E]\; f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I)\, dt_E\, dw_E\, dw_I \\
&\quad + \int_{C+s}^{r_I} \int_{C+s}^{w_I} \int_{C+s}^{w_E} [w_E - t_E]\; f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I)\, dt_E\, dw_E\, dw_I \\
&\quad + \int_{r_I}^{C+1-\alpha} \int_{C+s}^{r_E} \int_{C+s}^{w_E} [w_E - t_E]\; f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I)\, dt_E\, dw_E\, dw_I
\end{aligned}$$

(<u>case2</u>) $r_I > r_E$

$$\begin{aligned}
E[\Pi^2(t_I)] &= \int_{C-\alpha}^{r_E} \int_{\max[C+s,\,t_I]}^{r_E} [w_E - t_I]\; f_{E(1:N-1)}(w_E) f_I(t_I)\, dw_E\, dt_I \\
&\quad + \int_{C-\alpha}^{r_I} \int_{r_E}^{C+s+1} [r_I - t_I]\; f_{E(1:N-1)}(w_E) f_I(t_I)\, dw_E\, dt_I \\
E[\Pi^2(t_E)] &= \int_{C+s}^{r_E} \int_{t_I}^{C+s+1} \int_{C+s}^{w_I} [w_I - t_E]\, f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I)\, dt_E\, dw_E\, dw_I \\
&\quad + \int_{r_E}^{C+1-\alpha} \int_{r_E}^{C+s+1} \int_{C+s}^{r_E} [r_E - t_E]\; f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I)\, dt_E\, dw_E\, dw_I \\
&\quad + \int_{C+s}^{r_E} \int_{C+s}^{w_I} \int_{C+s}^{w_E} [w_E - t_E]\; f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I)\, dt_E\, dw_E\, dw_I \\
&\quad + \int_{r_E}^{C+1-\alpha} \int_{C+s}^{r_E} \int_{C+s}^{w_E} [w_E - t_E]\; f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I)\, dt_E\, dw_E\, dw_I
\end{aligned}$$

J.2 Buyer's expected cost

The buyer's objective is to find $r = (r^1,\, r_0^2,\, r_I^2,\, r_E^2)$ which minimizes the buyer's expected total cost.

$$\min_r ETC_{OPT}(r) = EC^1(r^1, r_I^2, r_E^2) + (1 - p_0(r^1))EC_1^2(r_I^2, r_E^2) + p_0(r^1)EC_0^2(r_0^2),$$

We show the explicit function of each expected cost in each period, EC^1, EC_0^2, and EC_1^2. We start with EC_0^2 since we can easily obtain the optimal r_0^2 from Lemma 6.1. The buyer's expected cost at $T = 2$, if there is no incumbent, is

$$\begin{aligned} EC_0^2 \quad = \quad & \int_{C+s}^{r_0^2} w\, f_{E(2:N)}(w)\, dw \\ & + N r_0^2 F(r_0^2)[1 - F_{E(1:N-1)}(r_0^2)] \\ & + (C + 1 + s)\, [1 - F_{E(1:N)}(r_0^2)] \end{aligned}$$

Facing the asymmetric bidders, if there is an incumbent, the buyer faces six cases for EC_1^2; (a) an incumbent wins, and an entrant defines the payment, (b) an incumbent wins, and he receives the reservation price r_I^2, (c) an entrant wins, and an incumbent defines, (d) an entrant wins, and he receives the reservation price r_E^2, (e) an entrant wins, and other entrant defines, and (f) there is no winners among N suppliers and the buyer incurs an opportunity cost. We assume that $C - \alpha \le r_I^2 \le C + 1 - \alpha$ and $C + s \le r_E^2 \le C + 1 + s$. The probability who wins and who defines the payment differs depending on the relative position of r_I^2 and r_E^2 as follows.

(<u>case1</u>) $r_I \le r_E$: (we drop superscript of 2)

$$\begin{aligned}
EC_1^2 \;=\; & \int_{C-\alpha}^{r_I} \int_{\max[C+s,\,t_I]}^{r_I} t_E\, f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I \\
& + \int_{C-\alpha}^{r_I} \int_{\max[r_I,C+s]}^{C+s+1} r_I\, f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I \\
& + \int_{C+s}^{r_I} \int_{t_I}^{C+s+1} \int_{C+s}^{t_I} (N-1) t_I f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \\
& + \int_{r_I}^{C+1-\alpha} \int_{r_E}^{C+s+1} \int_{C+s}^{r_E} (N-1) r_E\, f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \\
& + \int_{C+s}^{r_I} \int_{C+s}^{t_I} \int_{C+s}^{w_E} (N-1) w_E\, f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \\
& + \int_{r_I}^{C+1-\alpha} \int_{C+s}^{r_E} \int_{C+s}^{w_E} (N-1) w_E\, f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \\
& + \int_{r_I}^{C+1-\alpha} \int_{r_E}^{C+s+1} (C+1+s)\, f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I
\end{aligned}$$

(<u>case2</u>) $r_I > r_E$

$$\begin{aligned}
EC_1^2 \;=\; & \int_{C-\alpha}^{r_E} \int_{\max[C+s,\,t_I]}^{r_E} t_E\, f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I \\
& + \int_{C-\alpha}^{r_I} \int_{r_E}^{C+s+1} r_I\, f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I \\
& + \int_{C+s}^{r_E} \int_{t_I}^{C+s+1} \int_{C+s}^{t_I} (N-1) t_I f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \\
& + \int_{r_E}^{C+1-\alpha} \int_{r_E}^{C+s+1} \int_{C+s}^{r_E} (N-1) r_E\, f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \\
& + \int_{C+s}^{r_E} \int_{C+s}^{t_I} \int_{C+s}^{w_E} (N-1) w_E\, f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \\
& + \int_{r_E}^{C+1-\alpha} \int_{C+s}^{r_E} \int_{C+s}^{w_E} (N-1) w_E\, f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \\
& + \int_{r_I}^{C+1-\alpha} \int_{r_E}^{C+s+1} (C+1+s)\, f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I
\end{aligned}$$

The fifth and sixth integral terms are the case (e) in both $r_I \le r_E$ and $r_I > r_E$.

At $T = 1$, the buyer faces symmetric bidders with the bidding strategy of $B^1_{OPT}(t^1)$.

The buyer's expected cost at $T = 1$ is

$$
\begin{aligned}
EC^1 &= \int_C^{r^1} B^1_{OPT}(w)\, f_{(2:N)}(w)\, dw \\
&\quad + N B^1_{OPT}(r^1) F(r^1)[1 - F_{(1:N-1)}(r^1)] \\
&\quad + (C+1)\,[1 - F_{(1:N)}(r^1)]
\end{aligned}
$$

Note that $B^1_{OPT}(w^1) = w^1 - E[\Pi^2(w^2_I)] + E[\Pi^2_1(w^2_E)]$ ($p_0 = 0$) since $w^1 < r^1$, and the reservation payment is $B^1_{OPT}(r^1)$, not r^1 (the buyer's reserve price is based on the supplier type).

APPENDIX K

OPTIMAL INDEPENDENT SEQUENTIAL AUCTIONS

In chapter 6, we obtained the optimal reserve prices in sequential auctions. Since the basic idea is very similar to obtain the optimal independent sequential auctions, we study the optimal independent sequential auctions (as we can see later, this optimal independent sequential auctions is not the optimal mechanism in our model). Under a regularity condition that $t + \frac{F(t)}{f(t)}$ is a monotone strictly increasing function of t for all t, the optimal independent sequential auctions can be obtained by finding the appropriate reserve prices under Vickrery auctions. However, the buyer sets the reserve price in each period to minimize the buyer's expected cost in each period, which differs from OPT in section 6.1. We will see how to obtain the optimal independent sequential auctions and why it can not be the optimal mechanism in our model. Our work is based on Sorensen[40][1].

K.1 The Optimal Independent Sequential Auctions (OPT-SI)

In the design of the optimal mechanisms for independent sequential auctions, the buyer concerns only each period at one time, i.e., the optimal design of auction is restricted to minimize the each expected cost at each period. That is, at $T = 1$, the buyer focuses on the design of the optimal auction only for $T = 1$. At $T = 2$, the buyer designs the optimal auction for $T = 2$ given the outcome of the first optimal auction(i.e., allocation and the payment). Thus, the buyer designs the optimal auction only for the period which she currently faces. Since the second optimal auction depends on the allocation of the first optimal auction, we first characterize the second optimal auction.

Even if the auction designs at $T = 1$ and 2 are independent, the second auction design

[1]Levin[24] and Cripps and Ireland[10] also studies the optimal auctions for complementary goods.

depends on the outcome at $T = 1$:

- **no incumbent at** $T = 2$ The buyer does not purchase at $T = 1$ from N suppliers (symmetric bidders at $T = 2$)

- **an incumbent at** $T = 2$ The buyer purchases at $T = 1$ from one of N suppliers (asymmetric bidders at $T = 2$)

When there is no incumbent at $T = 2$, the buyer faces N symmetric bidders and this is the standard optimal procurement auction, i.e., the Vickrery auction with the optimal reserve price r_0^2,

$$r_0^2 + \frac{F_E(r_0^2)}{f_E(r_0^2)} = OC^2 = C + 1 + s \text{ (opportunity cost of the buyer)}$$

When there is an incumbent at $T = 2$, it is also the Vickrery auction with two different optimal reserve prices for an incumbent and $N - 1$ entrants, r_I^2 and r_E^2, respectively. The reserve prices are,

$$\begin{aligned} r_I^2 + \frac{F_I(r_I^2)}{f_I(r_I^2)} &= OC^2 = C + 1 + s \\ r_E^2 + \frac{F_E(r_E^2)}{f_E(r_E^2)} &= OC^2 = C + 1 + s, \end{aligned}$$

At $T = 1$, the buyer faces N symmetric bidders. This is also the standard optimal auction except that the bidder's reported type is same as his willingness to be paid at $T = 1$. That is, bidder t's reported type is below his cost by amount of the expected profit at $T = 2$ under the optimal auctions at $T = 2$. Given two cases at $T = 2$, bidder t's reported type is,

$$t - \underbrace{\left[[E[\Pi^2(t_I)] - \left((1 - p_0(t))E_1[\Pi^2(t_E)] + p_0(t)E_0[\Pi^2(t_E)]\right)\right]}_{(\equiv E\pi^2)},$$

where $p_0(t)$ is the probability that the bidder t faces $N - 1$ entrants (there is no incumbent at $T = 2$), $[E[\Pi^2(t_I)]$ is the expected profit as an incumbent of bidder t, $E_0[\Pi^2(t_E)]$ the expected profit as an entrant if there is no incumbent, $E_1[\Pi^2(t_E)]$ the expected profit as an entrant if there is an incumbent.

A bidder t's virtual type is

$$c(t) = t - E\pi^2 + \frac{F(t)}{f(t)}$$

The optimal auction is same as the Vicrery auction(since it is the symmetric case and $E\pi^2$ is a constant, the payment is simply the second order statistics of $t - E\pi^2$ if it is below the reserve price, r^1), with the optimal reserve price r^1.

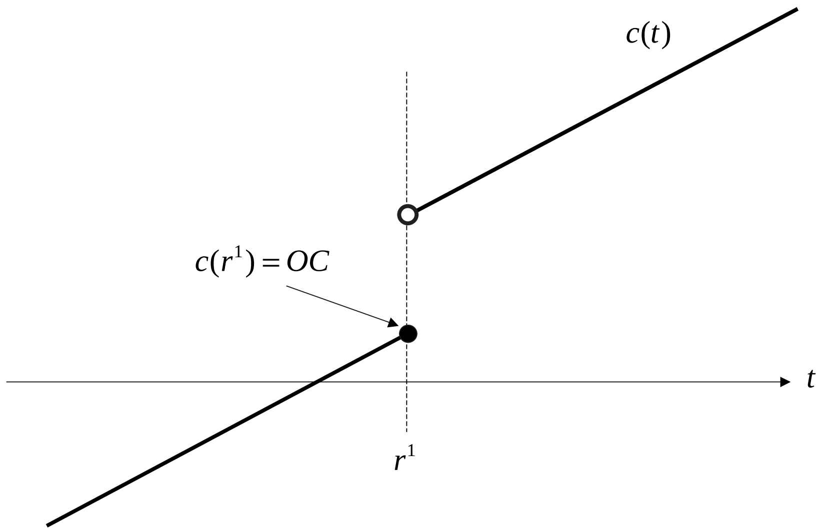


Figure K.1: t and the virtual type

The fact that there always exists an incumbent at $T = 2$ for a marginal bidder $t = r^1$(i.e., $p_0 = 0$), with combination that $c(t)$ is increasing in t (see figure K.1[2]), enables us to

[2]The virtual type of bidder t is

$$c(t) = \begin{cases} t - E[\Pi^2(t_I) \mid I] + E[\Pi^2(t_E) \mid 1] + \frac{F(t)}{f(t)}, & if\ t \leq r^1 \\ t - E\pi^2 + \frac{F(t)}{f(t)}, & if\ t > r^1 \end{cases}$$

$c(t)$ is increasing function since $c(t)$ if $t > r^1$ is greater than that if $t \leq r^1$ as follows.

$$\begin{aligned} & t - [E[\Pi^2(t_I) \mid I] + E[\Pi^2(t_E) \mid 1] + p_0 \left(E[\Pi^2(t_E) \mid 0] - E[\Pi^2(t_E) \mid 1]\right) + \frac{F(t)}{f(t)} \\ \geq\ & t - [E[\Pi^2(t_I) \mid I] + E[\Pi^2(t_E) \mid 1] + \frac{F(t)}{f(t)}\ (\because E[\Pi^2(t_E) \mid 0] \geq E[\Pi^2(t_E) \mid 1]) \end{aligned}$$

characterize the reserve price r^1 as follows.

$$r^1 - E[\Pi^2(t_I)] + E_1[\Pi^2(t_E)] + \frac{F(r^1)}{f(r^1)} = OC^1 = C + 1$$

K.2 Examples with Uniform Distribution

Under uniform distribution,

$$\begin{aligned} r_0^2 &= C + s + \frac{1}{2} && \text{(K.1)} \\ r_I^2 &= = C + \frac{1 + s - \alpha}{2} && \text{(K.2)} \\ r_E^2 &= = C + s + \frac{1}{2} && \text{(K.3)} \\ r^1 &= C + \frac{1}{2} + E[\Pi^2(t_I)] - E_1[\Pi^2(t_E)] && \text{(K.4)} \end{aligned}$$

Note that the buyer favors an incumbent, if there is an incumbent at $T = 2$:

$$Pr(t_I \leq r_I^2) = \frac{OC^2 - C + \alpha}{2} > \frac{OC^2 - C - s}{2} = Pr(t_E \leq r_E^2)$$

The inverse function of the virtual types of an incumbent and entrants are

$$\begin{aligned} c_I^{-1}(c_E(t_E)) &= t_E - \frac{\alpha + s}{2} && \text{(K.5)} \\ c_E^{-1}(c_I(t_I)) &= t_I + \frac{\alpha + s}{2} && \text{(K.6)} \end{aligned}$$

Given the reserve prices (equations (K.1)), the buyer's expected total cost at $T = 2$, if there is no incumbent is,

$$\begin{aligned} EC_0^2 &= \int_{C+s}^{r_0^2} w\, f_{E(2:N)}(w)\, dw \\ &+ Nr^2 F_E(r_0^2)[1 - F_{E(1:N-1)}(r_0^2) \\ &+ OC^2\, [1 - F_{E(1:N)}(r_0^2)] \end{aligned}$$

Given the reserve prices (equations (K.2) and (K.3)) and the inverse function of virtual costs of an incumbent and entrants (equations (K.5) and (K.6)), the buyer's expected total cost at $T = 2$, if there is an incumbent is,

$$
\begin{aligned}
EC_1^2 \quad = \quad & \underbrace{\int_{C-\alpha}^{\min[C+1-\alpha,\, r_I^2]} \int_{\max[C+s,\, t_I+\frac{\alpha+s}{2}]}^{r_I^2+\frac{\alpha+s}{2}} \left(t_E - \frac{\alpha+s}{2}\right) f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I}_{\left(c_I(t_I)<c_{E(1:N-1)}(t_E)<OC^2 \Rightarrow t_I<t_E-\frac{\alpha+s}{2}<r_I^2\right)} \\
+ \quad & \underbrace{\int_{C-\alpha}^{\min[C+1-\alpha,\, r_I^2]} \int_{r_I^2+\frac{\alpha+s}{2}}^{C+s+1} r_I^2\, f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I}_{\left(c_I(t_I)<OC^2<c_{E(1:N-1)}(t_E) \Rightarrow t_I<r_I^2<t_E-\frac{\alpha+s}{2}\right)} \\
+ \quad & \underbrace{\begin{aligned} &\int_{C+s-\frac{\alpha+s}{2}}^{\min[C+1-\alpha,\, r_E^2-\frac{\alpha+s}{2}]} \int_{t_I+\frac{\alpha+s}{2}}^{C+s+1} \int_{C+s}^{t_I+\frac{\alpha+s}{2}} \left(t_I + \frac{\alpha+s}{2}\right)(N-1) \times \\ &\qquad f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I \end{aligned}}_{\left(c_{E(1:N-1)}(t_E)<c_I(t_I)<c_{E(2:N-1)}(w_E)\,\&\, c_I(t_I)<OC^2 \Rightarrow t_E<t_I+\frac{\alpha+s}{2}<w_E\,\&\, t_I+\frac{\alpha+s}{2}<r_E^2\right)} \\
+ \quad & \underbrace{\int_{r_E^2-\frac{\alpha+s}{2}}^{C+1-\alpha} \int_{r_E^2}^{C+s+1} \int_{C+s}^{r_E^2} r_E^2\,(N-1) f_E(t_E) f_{E(1:N-2)}(w_E) f_I(t_I)\, dt_E\, dw_E\, dt_I}_{\left(c_{E(1:N-1)}(t_E)<OC^2<c_I(t_I)\,\&\, OC^2<c_{E(2:N-1)}(w_E) \Rightarrow t_E<r_E^2<t_I+\frac{\alpha+s}{2}\,\&\, r_E^2<w_E\right)} \\
+ \quad & \underbrace{\int_{C+s-\frac{\alpha+s}{2}}^{C+1-\alpha} \int_{C+s}^{\min[t_I+\frac{\alpha+s}{2}, r_E^2]} w_E\, f_{E(2:N-1)}(w_E) f_I(t_I)\, dt_E\, dt_I}_{\left(c_{E(1:N-1)}(t_E)<c_{E(2:N-1)}(w_E)<OC^2\,\&\, c_{E(2:N-1)}(w_E)<c_I(t_I) \Rightarrow t_E<w_E<r_E^2\,\&\, w_E<t_I+\frac{\alpha+s}{2}\right)} \\
+ \quad & \underbrace{\int_{r_I^2}^{C+1-\alpha} \int_{r_E^2}^{C+s+1} OC^2\, f_{E(1:N-1)}(t_E) f_I(t_I)\, dt_E\, dt_I}_{p_i(t_i)=0:\text{ no winning bidder}}
\end{aligned}
$$

Note that the payment to the winner is the second lowest virtual cost, not the second lowest cost as in the symmetric case.

Given the reserve prices (equations (K.4)) and the expected profit of bidder t at $T = 2$, the buyer's expected total cost at $T = 1$[3] is,

$$\begin{aligned} EC^1 &= \int_C^{r^1} (w - [E[\Pi^2(t_I)] + E_1[\Pi^2(t_E)]) \, f_{(2:N)}(w) \, dw \\ &+ N(r^1 - [E[\Pi^2(t_I)] + E_1[\Pi^2(t_E)]) F(r^1)[1 - F_{(1:N-1)}(r^1)] \\ &+ OC^1 \, [1 - F_{(1:N)}(r^1)] \end{aligned}$$

Hence, the buyer's expected total cost is a summation of the outcomes of two independent optimal auctions.

$$ETC = EC^1 + p_0 EC_0^2 + (1 - p_0) EC_1^2$$

The difference between the optimal independent sequential auctions(OPT-SI) and the optimal reserve price(OPT) in chapter 6 is clear. In the optimal independent sequential auctions, the buyer is only interested in minimizing her expected cost in each period. By this, the optimal reserve prices are simply obtained by Myerson[34] without the knowledge of the expected cost. So in optimal independent sequential auction, the buyer first find the optimal reserve prices, and then obtain the expected total cost given the optimal reserve prices. In contrast, under OPT in chapter 6, the reserve prices are correlated with each other, and the buyer finds the optimal reserve prices given the expected total cost.

As we will see, by the independence in current optimal auctions, the buyer fails to fully control over the bidding behavior at $T = 1$. This effect is substantial, so that even a simple SI can outperform OPT-SI. Table K.1 illustrates some examples.

[3] The expected profits of bidder t as an incumbent and an entrant at $T = 2$ are

$$\begin{aligned} E[\Pi^2(t_I)] &= \int_{C-\alpha}^{\min[C+1-\alpha,\, r_I^2]} \int_{\max[C+s,\, t_I + \frac{\alpha+s}{2}]}^{r_I^2 + \frac{\alpha+s}{2}} \left(w_E - \frac{\alpha+s}{2} - t_I\right) f_{E(1:N-1)}(w_E) f_I(t_I) \, dw_E \, dt_I \\ &+ \int_{C-\alpha}^{\min[C+1-\alpha,\, r_I^2]} \int_{r_I^2 + \frac{\alpha+s}{2}}^{C+s+1} (r_I^2 - t_I) \, f_{E(1:N-1)}(w_E) f_I(t_I) \, dw_E \, dt_I \\ E_1[\Pi^2(t_E)] &= \int_{C+s-\frac{\alpha+s}{2}}^{\min[C+1-\alpha,\, r_E^2 - \frac{\alpha+s}{2}]} \int_{w_I + \frac{\alpha+s}{2}}^{C+s+1} \int_{C+s}^{w_I + \frac{\alpha+s}{2}} \left(w_I + \frac{\alpha+s}{2} - t_E\right) \times \\ &\qquad f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I) \, dt_E \, dw_E \, dw_I \\ &+ \int_{r_E^2 - \frac{\alpha+s}{2}}^{C+1-\alpha} \int_{r_E^2}^{C+s+1} \int_{C+s}^{r_E^2} (r_E^2 - t_E) f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I) \, dt_E \, dw_E \, dw_I \\ &+ \int_{C+s-\frac{\alpha+s}{2}}^{C+1-\alpha} \int_{C+s}^{\min[w_I + \frac{\alpha+s}{2}, r_E^2]} \int_{C+s}^{w_E} (w_E - t_E) f_E(t_E) f_{E(1:N-2)}(w_E) f_I(w_I) \, dt_E \, dw_E \, dw_I \end{aligned}$$

Table K.1: Comparisons between the OPT-SI and SI

Market			Reserve Prices				Comparison of Expected Costs				
N	α	s	r^1	r_I^2	r_E^2, r_0^2	p_0	Δ^1	Δ_0^2	Δ_1^2	Δ^2	Δ
4	0.2	0.1	1.527	1.450	1.600	0.050	-6.3	-4.4	2.0	1.7	-2.0
6			1.520	1.450	1.600	0.012	-5.2	-3.6	1.8	1.7	-1.6
8			1.516	1.450	1.600	0.003	-4.9	-2.8	1.8	1.8	-1.3
10			1.514	1.450	1.600	0.001	-4.5	-2.4	1.8	1.8	-1.2
20			1.510	1.450	1.600	0.000	-4.0	-1.2	1.9	1.9	-0.9
4	0.05	0.1	1.512	1.525	1.600	0.057	-1.9	-1.8	1.1	0.9	-0.4
	0.1		1.517	1.500	1.600	0.054	-3.1	-2.6	1.4	1.2	-0.8
	0.2		1.527	1.450	1.600	0.050	-6.3	-4.4	2.0	1.7	-2.0
	0.3		1.539	1.400	1.600	0.045	-9.9	-6.4	3.3	2.8	-3.0
	0.4		1.552	1.350	1.600	0.040	-14.0	-8.6	4.6	4.0	-4.1
	0.5		1.566	1.300	1.600	0.035	-18.6	-11.1	6.0	5.4	-5.2
4	0.2	0.05	1.522	1.425	1.550	0.052	-4.5	-3.6	1.7	1.4	-1.4
		0.1	1.527	1.450	1.600	0.050	-6.3	-4.4	2.0	1.7	-2.0
		0.2	1.539	1.500	1.700	0.045	-9.9	-5.9	3.0	2.6	-2.9
		0.3	1.552	1.550	1.800	0.040	-14.2	-7.5	4.0	3.5	-3.9
		0.4	1.567	1.600	1.900	0.035	-18.5	-9.1	5.0	4.5	-4.6
		0.5	1.581	1.650	2.000	0.031	-23.0	-11.0	5.8	5.2	-5.2

$\Delta^1 = \frac{EC_{SI}^1 - EC_{OPT-SI}^1}{EC_{OPT-SI}^1}(\%)$, $\Delta_0^2 = \frac{EC_{SI}^2 - EC_{0(OPT-SI)}^2}{EC_{0(OPT-SI)}^2}(\%)$, $\Delta_1^2 = \frac{EC_{SI}^2 - EC_{1(OPT-SI)}^2}{EC_{1(OPT-SI)}^2}(\%)$,
$\Delta^2 = \frac{EC_{SI}^2 - EC_{(OPT-SI)}^2}{EC_{(OPT-SI)}^2}(\%)$, $\Delta = \frac{ETC_{SI} - ETC_{(OPT-SI)}}{ETC_{(OPT-SI)}}(\%)$

The expected cost at $T = 2$, if there is an incumbent, EC_1^2 is always lower than the expected cost at $T = 2$ under SI (see Δ_1^2 in table K.1). This is straightforward in the sense that at $T = 2$ OPT-SI and SI are exactly same, and under OPT-SI, the buyer finds the "optimal" reserve prices.

The expected cost at $T = 2$, if there is no incumbent, EC_0^2 would be greater (or smaller) than the expected cost at $T = 2$ under SI (see Δ_0^2 in table K.1). under OPT-SI, the buyer faces N symmetric bidders with cost distribution of $[C + s, C + 1 + s]$. When α and s are small, OPT-SI and SI would be very similar (if $\alpha = s = 0$, both become identical), and hence EC_0^2 is less than the expected cost under SI (e.g. when $N = 3$, $\alpha = s = 0.01$,

$\Delta_0^2 = 1.6$). However, with increasing α and s, the incumbent has lower cost distribution than entrant's under SI, and hence SI would take advantage of this.

The expected cost at $T = 1$ EC^1 would be greater (or smaller) than the expected cost at $T = 1$ under SI. When α and s are small, EC^1 would be smaller than the expected cost under SI (e.g., when $N = 3$, $\alpha = s = 0.01$, $\Delta^1 = 0.7$). However, as α and s increases, it is no longer the case. This is because the reported cost under OPT-SI is greater than the bid under SI. Bidders under OPT-SI at $T = 1$ should take into account the possibility that there would be no winner at $T = 1$.

Overall, OPT-SI does not actually give the optimal solution to the buyer in our model. This is because that the buyer designs the optimal auction only for each period. In this case, the optimal auction benefits the buyer at $T = 2$ because the buyer always designs the optimal auction according to the outcome at $T = 1$. However, at $T = 1$, the bidder's reporting cost (virtual cost) is actually greater than the bid under SI - the bidder does not fully discount his cost under OPT-SI because with positive probability, the buyer does not purchase at $T = 1$[4].

[4]As further step, we can change the model slightly :

- the buyer wants to minimizes her expected total cost at $T = 1$ and 2 as a whole. However, she does not control the second period beforehand. Instead, she finds the optimal reserve price at $T = 1$ which minimizes the expected total cost at $T = 1$ and 2 (ETC). That is, r^2, r_I^2, and r_E^2 are same as under OPT-SI.
- the buyer commits the reserve price at $T = 2$ in the first auction. The buyer tries to control the bidder's reported cost(virtual cost) at $T = 1$ by announcing the reserve prices at $T = 2$ in the first period (in above design, the buyer could not control the bidder's virtual cost at $T = 1$). Thus, the buyer finds the optimal reserve prices at $T = 1$ and 2 at the same time which minimize the buyer's expected total cost at $T = 1$ and 2.

The optimal reserve price r^1 in the first case would result; (a) the buyer's expected cost at $T = 1$ (EC^1) would be greater than that under OPT-SI, but (b) the buyer's expected cost at $T = 2$ ($p_0 EC_0^2 + (1-p_0)EC_1^2$) would be less than that under OPT-SI. Especially, (b) would dominate (a) effect which makes the buyer better off from OPT-SI. The last case would results the better outcome because the buyer chooses the optimal reserve prices r_I^2, r_E^2 and r^1 by considering to minimize ETC. However, it still does not give the better performance than other procurement mechanisms.

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